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Capacity Expansion in the Twin Cities: The Roads-Transit Balance

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16. Abstract (Limit: 200 words)

"What would it take to build our way out of congestion in the Twin Cities?" was the question posed by researchers five years ago. This previous study solved a roads-only network design problem (NDP) for the Twin Cities of Minnesota. Building on that work, another network design problem is examined for the Twin Cities metropolitan area of 3 million, to examine the tradeoff between demand side reductions and the limited access capacity expansion necessary to achieve desired levels of service. The problem is simplified by pre-determining a mode split, which allows for incorporating decreasing demand directly as an input rather than in the model formulation. The problem is solved using Sequential Linear Expansion (SLIE), a modified method of successive averages (MSA). Computation time for the large network is decreased to a reasonable length using another modification, the MSA with decreasing re-initialization (MSADR). A typical personal computer can solve this large-sized problem within 24 hours. For forecasted travel demand for 2030, it was found that if the number of trips were reduced by 20%, lane-miles needed to achieve LOS D decreases by up to 43%.

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Capacity Expansion in the Twin Cities: The Roads-Transit Balance

Final Report

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EXECUTIVE SUMMARY

Traffic congestion and what to do about it is seemingly a source of much discussion in many households, and in political and technical circles. As a community concern, it continues to rank as a top issue, as shown by its standing in many polls. A survey of residents of the Minneapolis-St. Paul metropolitan area conducted by the Metropolitan Council *(1)* found that for the fifth consecutive year, more residents perceive congestion as more important than any other concern, including crime, education, and housing. Discussion as to what to do about traffic congestion can be reactive in nature, and does not take into account the complicated nature of driver behavior and the ability of drivers to shift their routes, times and perhaps even the amount they travel. How to fund expanded and new roads to relieve congestion seems to be the center of much discussion, and so exploring what it would take to provide this better peak period travel for drivers seems worth further exploration, as it may become the ***de facto*** policy. The 2002 study "Building Our Way Out of Congestion" investigated this question of providing free flow travel on limited access roadways (expressways and freeways) for a system of single occupancy vehicles. This earlier study found that 1,146 lane-miles (1,844 lane-km) would need to be added to the existing limited access links to meet demand in 2020 while guaranteeing a level of service (LOS) C, which would be an increase in lane-miles of about 70% over the existing highway system. (Level of service varies from "A" (free flow), to "F" (breakdown of flow), with flow at LOS "E" at or near capacity.)

An obvious response to relieve traffic congestion would be adding supply to the road network, but highway lanes are not the only form of supply in ground transportation, and adjusting supply is not the only way to make supply and demand reach equilibrium. This research report extends the 2002 study *(3)* by investigating the effect transit and other methods of reducing single occupancy vehicle auto demand might have on the amount of capacity expansion that would be needed to meet travelers' expectations of uncongested driving conditions. While fewer vehicles obviously can mean less required expansion, the relationship is not linear. Exploring the tradeoff of transferring trips onto other modes provides an interesting insight into the potential for providing for travel through means other than straight capacity expansion, and provides an additional tool in assessing alternatives to building our way out of congestion.

A number of formulations of the multimodal network design problem (MNDP) were considered for this problem. However, to examine the tradeoff between capacity expansion and alternative mode use, mode split was pre-determined (to give us a variety of points for a trend line) rather than calculated, as is generally the case in travel demand models. Details of the alternative modes were not specified, as alternative modes or demand reduction measures can come in many forms. A number of solutions were examined to bound the problem, from using assumptions that did not load any trips back onto the congested roadways (simulating straight travel demand reduction) to those that did (simulating increased bus transit usage with shared right of way). The problem could, therefore, be formulated nearly identically to the previous work, Building Our Way Out of Congestion. Minor changes to the formulation result from changes in the Twin Cities regional framework. This treatment of transit avoids the complications and difficulties solving an MNDP.

Of importance to the project was the capability of the solution procedure to be implemented on a desktop computer, so that planners could conduct additional analyses. This resulted in an algorithm that modified the method of successive averages (MSA), dubbed Sequential Linear Expansion (SLIE). However, a significant increase in travel demand from the 2020 to 2030 forecast created difficulties in convergence for the solution procedure. New solution procedures were reviewed to find a procedure that would converge more efficiently without requiring complicated revisions to the previously developed routine. The review led to a modified form of the MSA that is slightly revised to include re-initialization, called MSA with decreasing re-initialization, or MSADR. With the 2030 inputs, 30-45 iterations were needed to converge, with 21 minutes per iteration on a 3.0GHZ Pentium 4 processor with 1GB RAM.

The Twin Cites NDP was solved for several different scenarios, to account for the different types of interactions between the transit vehicles and the road network. Solutions were found for seven pre-determined mode split values, ranging from 0 to 20 percent, for each scenario. For one bound, it was assumed that there was no interaction between the shifted trips and the road network. This minimum interaction scenario could represent situations such as a grade-separated transit system, ride-sharing, or a decrease in trips demanded, for instance due to telecommuting. Next, it was assumed that all of the transit trips between an origin-destination pair were made by standard bus traveling non-stop on the road network between origin and destination. This formulation of transit is unrealistic, but provides a bound to the problem representing the maximum effect that transit could have on the road network. Finally, it was assumed that alternative mode trips between an origin-destination pair were aggregated to a district level, and then made by a standard bus traveling non-stop, akin to express buses. Additionally, while these formulations were tailored for transit, they were chosen because they were flexible enough to be applied to many other demand-side measures, such as telecommuting or job/home relocation. The effects of removing 20% of the demand by having 20% of the workforce work from home can be directly represented by the first bound. Effects of all demand-reduction measures should fall within the bounds set in this problem. Solutions were found to be very sensitive to the demand formulation used. For example, with 20% of total demand attributed to alternative modes or demand reduction, the expansion required to achieve LOS D ranged from 3.9% to 42.6% less than that required to serve 100% SOV demand.

For each scenario, every mode split point was solved for both LOS C and LOS D constraints to provide sensitivity analysis for level of service. The choice of service level turned out to make a significant difference in capacity expansion necessary. With no demand reduction, to provide LOS C required 1,041, or 66%, more lane-miles (1,675 lane-km) compared to LOS D. With 20% demand reduction, LOS C required 67% to 86% more lane-miles over LOS D for the three different scenarios. However, each level of service showed similar sensitivity to each of the different transit formulations.

If the Twin Cities were to reduce SOV demand by 20%, LOS D could be achieved on all limited access roads with 36% percent less expansion than needed if all trips were made by SOV. If, distributed evenly throughout the work week, each worker traveling by SOV made one work trip per week by transit, or eliminated it by telecommuting, (a 20% reduction in demand), 1,013 lane-miles, instead of 1,578 (a reduction of 36%), would be need to provide him or her with LOS D conditions the other four days of the week.

Chapter 1: INTRODUCTION

A survey of residents of the Minneapolis-St. Paul metropolitan area found that traffic congestion was the "single most important problem" in 2005. *(1)* For the fifth consecutive year, more residents perceived congestion as more important than any other concern, including crime, education, and housing. Favored solutions from the survey included both commuter/light rail transit and adding more lanes to freeways. Evidence of both is all around. The Metropolitan Council, the region's Metropolitan Planning Organization, lists doubling transit ridership as one of its goals in the 2030 Transportation Policy Plan *(2)*, and planning continues on several rail lines around the region. Capacity is being added to major regional Interstates 94, 494, and 394. However, all projects, whether they primarily benefit automobiles or transit, must compete for limited funds. Costs and benefits of every aspect of the Central Corridor transit line are under scrutiny to find somewhere to cut costs. The untangling of the Interstate 35W and State Highway 62 commons is on hold until financing can be secured. Projects fight for insufficient funds and money is allocated without a good sense of their interaction.

In step with this social environment, this project follows up on a 2002 study, Building Our Way Out of Congestion (BOWOC), in which the Twin Cities continuous network design problem (NDP) was solved for the road network. *(3)* That study focused on roads to determine what it would take to build roads to satisfy uncongested travel demand on freeways, assuming all travel was by single occupancy vehicle (SOV). The study found that 1,146 lane-miles (1,844 lane-km) would need to be added to the existing limited access links to meet demand in 2020 at level of service (LOS) C, an increase of about 70% over the existing highway system.

While adding supply to the road network to relieve road congestion may be a natural response to the problem, automobiles are not the only form of supply in ground transportation, and adjusting supply is not the only way to make supply and demand meet. This project investigates what effect transit, a form of transportation supply which can decrease vehicular demand, could have on congestion and the capacity expansion needed to accommodate it. The results of this project can also be applied to alternate methods of decreasing travel demand such as ridesharing or telecommuting. Knowing the effectiveness of alternate modes at reducing required capacity expansion would provide an additional tool in assessing the costs and benefits of the alternatives to building our way out of congestion.

Originally, the concept was to incorporate transit usage into the problem, which resulted in searches for multimodal network design problem (MNDP) and related formulations. However, further discussion and thought determined that these were inapplicable to the problem and instead a demand-side manipulation was used. Instead of applying transit via a transit network, transit was integrated through a reduction in travel demand, which is measured in trips that are assumed to be made by SOV. This is discussed in Chapter 2.

In addition to updating the 2002 study with the inclusion of transit, two other goals were central to this project. The first was the capability of the solution procedure to be implemented on a standard personal computer. A desired product of this project was a program that would utilize only tools available to transportation planners so that planners could evaluate scenarios of interest not covered in this report. The problem formulation incorporates the network and demand forecasts produced by the Metropolitan Council and is outlined in Chapter 3, along with the sequential linear expansion (SLIE) solution procedure, a modified form of the method of success averages (MSA). Implementation details and results are presented in Chapter 4.

The third main consideration was another aspect of the original study that deserved further consideration, the cost formulation. A point for further research from the original project was to determine if there were any reliable estimates for the dollar cost of expansions. At the time, no reliable formulations were available and so costs were not included in the problem. The objective function simply minimized the number of lane-miles to be added. An update of cost formulations is given in Chapter 5, although the same problems remain and cost is not included as an outcome of this project either. Chapter 6 summarizes the conclusions and offers possibilities for further research.

Chapter 2: NETWORK DESIGN PROBLEM FORMULATIONS

Given unlimited land and financial resources, it would be simple to provide enough roadways to accommodate travel demand – an additional 15 lanes everywhere should suffice. As discussed in the introduction, though, resources are not unlimited, and instead quite low. Because of this, it is necessary to determine the minimal network expansion that accomplishes the goal of relieving congestion. This is the essence of the network design problem (NDP) – determining the most efficient changes to a network that are effective in achieving the stated goals. The NDP encompasses a class of optimization problems modeling this process and a substantial body of research explores the various different formulations this problem can take, along with associated solution procedures. A review of other NDP formulations can be found in BOWOC *(3)*.

One classification of NDPs is the distinction between continuous and discrete. In a continuous problem, changes are made only to the attributes of the existing network. The topology of the network remains the same, but, for instance, links can be widened. With the discrete NDP, changes are made to the network's physical configuration. Most commonly in transportation this would involve the addition of links. In a developed area like the Twin Cities, changes to the road network configuration are unlikely, establishing this problem as a continuous NDP. Additionally, the only changes considered are the addition of lanes to the existing links. There will be no removal of lanes or other adjustments to change the capacity of links.

As in BOWOC, the demand side of the problem models only the route assignment and not travel demand. Demand is taken to be fixed, in part to make use of the Metropolitan Council's demand forecasts and in part because of the lack of a reasonable solution algorithm for a problem of this large scale. It is possible that decrease in congestion, caused by an increase in supply or transit usage, would induce more travel demand over time, but that would only increase the already overwhelming expansion necessary. The model used for the demand side is a stochastic user equilibrium (SUE), in which, at the solution, no user can change routes to decrease their own travel time, with allowance for variation in users' perceptions.

Multimodal Network Design Problem

Traditionally, NDPs have dealt with the expansion of the road network, although Transit Network Design Problems (TNDP) and Multimodal Network Design Problems (MNDP) have been addressed more recently. Van Nes *(4)* provides guidelines on the formulations for combinations of modes based on hierarchical network structures. Other recent work that is relevant deals with Network Pricing Optimization (NPO). NPOs are closely related to NDPs, but rather than determining optimal supply expansions the design variables are tolls. Ferrari *(5)* presents a bimodal transport system and proposed new methodology for solving the programming problem with equilibrium constraints and applies this to a medium sized network. For this problem, the desired formulation would attempt to model a more realistic process of planning efforts and satisfying travelers' expectations, and it would also solve a problem for a network of an order of magnitude larger than those presented.

In this problem, the focus is on only a portion of the MNDP. Mode split, the decision of which mode will be used to make each trip, does not need to be solved for within the solution algorithm. Generally, mode split is chosen based on a number of factors, including travel time and other costs. To calculate these costs greatly complicates the problem, as it would require sensitivity adjustments to travel time and pricing in order to produce the mode split percentages desired and would be unnecessary in this study. In this model, the desired result was to see what effect different levels of demand would have on the amount of expansion necessary to provide given levels of service on the limited access roads. To examine this tradeoff between reducing SOV demand and capacity expansion, pre-determined levels of demand could be applied to provide data points for the trade off trendline.

At the outset, the 2030 model transit network from the Metropolitan Council was to be used for carrying the transit demand. However, this presented too many difficulties, including determining transit's interactions with the road network and coordination of the networks. Instead, it was decided that not specifying a transit network broadens the applicability of the project results. Not specifying a transit network allows for examination of various modes of transit, from those that share rights-of-way with automobiles to the grade-separated. This flexibility allows not only for analysis of different modes of transit, but also for other SOV demand-reducing measures such as telecommuting and job or home relocation.

Without using a transit network model, the question became how to distribute transit use through the network. An attempt to use proportions from a transit O-D demand forecast resulted in negative SOV demand for some O-D pairs. The idea was to distribute the pre-determined total transit demand (found by taking a predetermined percentage of the total travel demand) to the O-D pairs according to the portion of transit demand each O-D pair was forecasted to have. For example, if there were a total of 400 transit trips to distribute and only 100 transit trips total in the transit demand forecast and one O-D pair had been forecasted to have 25 trips, then that O-D pair would be assigned 100 transit trips (a quarter of the total transit trips). If that O-D pair has less than 100 trips forecasted in the travel demand forecast, then the reducing the demand on that O-D pair by 100 trips results in a negative number of trips for that O-D pair.

The final solution was to designate a percentage of trips as single occupancy vehicle trips uniformly across the trip table, later accounting for different types of congestion interactions on the road network. With large differences in transit convenience and travel time cost within the metro area, uniform distribution does not model transit demand as would be expected. However, one might expect that if transit were focused more on the congested areas, it could be both more efficient and more effective.

With this treatment of transit, the problem could be formulated nearly identically to the previous work, Building Our Way Out of Congestion. Minor changes to the formulation result from changes in the Twin Cities regional framework.

Transit Formulation

A number of different possible formulations for transit were explored, each with different effects on the road system. For example, to reduce demand the number of trips made can be reduced or the number of vehicles on the roads can be reduced. The number of vehicles can be reduced by having trip makers use various forms of transit or other alternate modes, each of which has different effects on traffic on the road. Grade-separated forms of transit such as subways and personal rapid transit (PRT), greater ridesharing, or even bicycle use are trips that have no significant interaction with the road network. These forms have an effect similar to reducing trip demand altogether, which can also be achieved by demand management techniques such as telecommuting. At-grade modes can share the road with personal vehicles, like buses and carpools, or they might have their own right-of-way but still intersect roads. For each alternate mode form, the effects would have to be determined individually.

To account for these various alternate modes, rather than attempt to solve every one individually, formulations were chosen that would represent the maximum and minimum effects this could have on limited access capacity expansion, thereby effectively bounding the problem. This allows policymakers to see the variation in results that the choice of transit mode, or other SOV demand reduction, can make. It also avoids using specific forecasts of alternate modes which would complicate the computations.

The maximum effect that transit can have is represented by the situation where demand is essentially removed from the road network. Pre-determined percentages of the demand for each O-D pair are removed, and the reduced demand is used to solve the problem. This formulation can represent a decrease in trips demanded as well as trips made by grade-separated transit options. Alternatively, this solution can be viewed as the solution for other types of transit if the demand numbers include the transit vehicles that would be added back onto the network.

A minimal effect solution is represented by the situation where trips are added back onto the network as transit vehicles. The transit vehicles were thought of as standard buses for purposes of determining occupancy and passenger car equivalency (PCE). Occupancy of 50 and PCE of 2.5 were used. However, the transit vehicles are assumed to travel directly from origin to destination and not stopping to accommodate trips along the way. Trips that start in the same TAZ and end in different TAZs lying along the same path would not be made with the same vehicle, even if the vehicle's occupancy was low compared to its maximum occupancy. This would provide maximum convenience for users and allow for growth in transit ridership, but would be very inefficient. The inefficiency was mitigated a little by requiring at least 2.5 trips be made by transit before a transit vehicle would operate between an O-D pair. If fewer than 2.5 transit trips were demanded, these trips would still have to be made by SOV.

In addition to the two bounds, the problem was solved for a third formulation, one that would provide an idea of where within the bounds a more realistic transit system might fall. This was done by aggregating the transit trips into transportation analysis districts (TADs) to emulate express buses. Trips would still be decreased by pre-determined percentages according to TAZs, but those transit trips would then be aggregated by TADs, allowing for fewer and fuller transit vehicles. This formulation has the benefits of not requiring much additional infrastructure, being similar to what currently exists in the Twin Cities, and focusing on the trips that occur on the links of interest – the limited access highways. While TADs are not officially used by the Metropolitan Council any longer, an unofficial TAD system based on the last official version (from the 1980s) was used.

Chapter 3: SOLUTION APPROACH

Problem Formulation

As is standard, a roadway system can be represented as a network consisting of a set of nodes, connected by directed links. A sub-set of these nodes, called centroids, act as points of origin and destination for travel to and from zones. The total travel demand between each combination of origin and destination (O-D) zones is specified for some planning interval (such as the morning peak hour), and quantified in the form of an O-D matrix, which is assumed to be known. The O-D pairs in the network are indexed by $j = 1,\ldots,m$ and d_j denotes the demand between O-D pair j during the planning interval. The road network consists of n links in the network, indexed $k = 1,\ldots,n$. Associated with each of these links are three quantities: x_k denoting the demand traffic flow on link k, y_k denoting the proposed expansion of k's capacity, and z_k denoting the current capacity of link k. Thus the capacity after expansion of link k would be the sum of y_k and z_k. Each link also has a user cost function, $c_k(x_k,z_k+y_k)$, which describes the cost of traversing that link.

Since it was determined that expansion cost estimates could not be used here because current available estimation methods are not reliable, the optimization criterion chosen was the same function as used in Building Our Way Out of Congestion

$$\left(\sum_k l_k y_k\right) \tag{1}$$

where l_k is the length of link k, and y_k is the variable associated with capacity expansion for link k. Using a default value for lane capacity this measure can be readily converted to total lane-miles of new capacity, and this was a quantity that could be readily appreciated lay audiences. The solution could also be used in an off-line computation of cost when (and if) defensible cost functions become available.

Ensuring that network enhancements are the only valid changes to existing capacity, a non-negativity constraint is placed on the expansion variable

$$y_k \geq 0 \qquad \text{for links that are candidates for expansion} \tag{2}$$

$$y_k = 0 \qquad \text{for non-candidate links}$$

To account for the responses of highway users to the capacity changes, stochastic user equilibrium (SUE) conditions were used to determine equilibrium traffic flows. As has been detailed in Sheffi *(6)*, the SUE conditions can be expressed as a system of nonlinear equality constraints of the form

$$x_k = \sum_{j=1}^{m} d_j q_{jk} \qquad \text{k} = 1,\ldots,\text{n} \tag{3}$$

where q_{jk} denotes the probability that a trip between O-D pair j uses link k. If we let r index the set of feasible routes on our network and p_{jr} denote the probability a trip between O-D pair j uses route r, these link use probabilities depend the route choice probabilities

$$q_{jk} = \sum_r d_{jrk}\, p_{jr} \tag{4}$$

where

$$d_{jrk} = \begin{cases} 1 & \textit{if link k lies on route r between O - D pair } j \\ 0 & \textit{otherwise} \end{cases} \tag{5}$$

An explicit function for the route choice probabilities can be found using logit route choice

$$p_{jr} = \frac{e^{-qc_{jr}}}{\sum_s e^{-qc_{js}}} \tag{6}$$

where q is a parameter which is determined by the variance of the route cost perception errors and c_{jr} is the cost of traversing route r between O-D pair j. Assuming that the path cost is simply the sum of the corresponding links costs, then

$$c_{jr} = \sum_{k=1}^{n} d_{jrk}\, c_k\left(x_k, y_k + z_k\right) \tag{7}$$

where c_k denotes the travel cost on link k. To allow for congestion, travel cost on a link is taken to be an increasing function of the demand on that link. The most recent Twin Cities transportation planning model uses Spiess's conical volume delay function

$$c_k\left(x_k, y_k + z_k\right) = c_k^0\left(2 + \sqrt{\left(b_k \times \left(1 - \frac{x_k}{y_k + z_k}\right)^2 + a_k\right)} - d_k \times \left(1 - \frac{x_k}{y_k + z_k}\right) - f_k\right) \tag{8}$$

where c_k^0 is the free-flow travel cost on link k and b_k, a_k, d_k, and f_k are coefficients that are calibrated by road class. (*7*) TABLE 3.1 shows the values of the coefficients for each road type, while FIGURE 3.1 compares the travel time as a function of the volume to capacity ratio for each of the conical volume delay functions and the Bureau of Public Roads (BPR) function.

TABLE 3.1 Delay Function Coefficients for Each Road Class

Road Class	b_k	a_k	d_k	f_k
1-Metered Freeway	16	1.361	4	1.167
2-Unmetered Freeway	16	1.361	4	1.167
3-Metered Ramp	16	1.361	4	1.167
4-Unmetered Ramp	16	1.361	4	1.167
5-Divided Arterial	16	1.361	4	1.167
6-Undivided Arterial	25	1.266	5	1.125
7-Collector	36	1.210	6	1.100
8-HOV Ramp	16	1.361	4	1.167
9-Centroid Connector	0	0	0	1
0-HOV facility	16	1.361	4	1.167

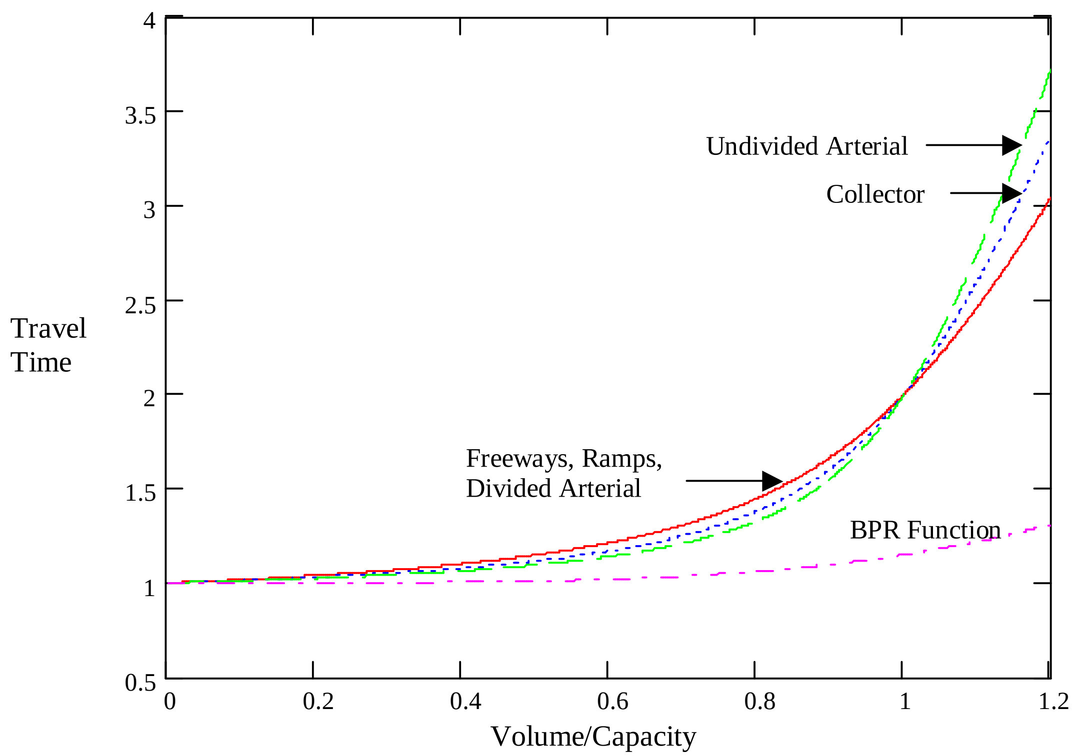

FIGURE 3.1 Comparison of Delay Functions for t_0=1

One way to specify level of service targets would be to place upper bounds on the average user cost functions. That is

$$c_k(x_k, z_k + y_k) \le \hat{c}_k \tag{9}$$

on some or all of the network's links. When user cost depends is a monotonically increasing function of the volume to capacity (V/C) ratio, these constraints have an equivalent expression as bounds on the V/C ratio

$$\frac{x_k}{y_k + z_k} \le \tilde{c}_k \tag{10}$$

Collecting together the components of this optimization problem, and letting **x** denote the n-dimensional vector containing the link demand flows, gives our variant of the NDP.

NDP*:

$$\text{minimize}\left(\sum_k l_k y_k\right) \tag{11}$$

subject to

$$x_k = \Sigma_j\, d_j\, q_{jk}(\mathbf{x}) \qquad k=1,..,n \tag{12}$$

$$x_k - \tilde{c}_k y_k \le \tilde{c}_k z_k \qquad \text{for candidate links for expansion} \tag{13}$$

$$y_k \ge 0 \tag{14}$$

With the exception of the volume delay function, the formulation is identical to the problem formulation in Building Our Way Out of Congestion.

Solution Procedure

The problem of solving network design problems (NDP) is one that many researchers have investigated and one of the primary issues surrounding them is the expense of the solution procedure, in terms of both time and computer power. Davis *(8)* posed a differentiable and tractable version of the NDP by employing SUE constraints, enabling standard algorithms for solving non-linear programs to be used. The computation of the minimum of a differentiable objective function, subject to a set of differentiable constraints, is a problem for which numerous solution algorithms are available. In BOWOC, the optimization program MINOS *(9)* was selected for implementation as the NDP solution procedure because of its reported success, across a variety of disciplines, in solving large, non-linear programming problems. However, while MINOS successfully solved the NDP for smaller test problems, a successful implementation for a network representing a metropolitan region was not accomplished. When alternate methods were investigated in an attempt to circumvent the large Jacobian difficulties, it became apparent that a modified solution procedure could be constructed to take advantage of the specific formulation of this NDP. This procedure uses the method of successive averages (MSA), but does not require any derivative information, which considerably simplifies computational requirements. The new step incorporated into the MSA that enabled the problem to be solved was called Sequential LInear Expansion (SLIE).

As described in BOWOC, the steps of the procedure are as follows:

Step 0: Initialization. Perform a stochastic network loading based on a set of initial travel costs (c_k^0), generating a set of link flows (x_k^0). Set counter N to 1.

Step 1: For each link k, set $y_k^{N+1} = \max\left(0, \frac{x_k^N}{\tilde{c}_k} - z_k\right)$

Step 2: Update travel costs for each link based on current link flows and capacity, $c_k^{N+1}\left(x_k^N, z_k + y_k^{N+1}\right)$, then find shortest paths, and rank the nodes according to increasing cost.

Step 3: Perform a stochastic network loading assignment procedure based on the current set of link travel costs ($c_k^{N+1}\left(x_k^N, z_k + y_k^{N+1}\right)$), yielding the auxiliary link flow pattern (x_k^{*N}), using Dial's Algorithm.

Step 4: Find the new flow pattern by setting $x_k^{N+1} = x_k^N + \frac{\left(x_k^{*N} - x_k^N\right)}{N}$

Step 5: If converged, then stop. If not increment counter N and go back to step 1.

A proof that this algorithm converges to Karush-Kuhn-Tucker points of NDP* is given in Appendix A.

For this study, the optimization solution procedure was modified slightly to expedite convergence. The 39% demand increase, and resulting congestion increase, between the 2020 forecast and the 2030 forecast was significant enough to create difficulties reaching convergence using the SLIE routine. Cascetta and Postorino's *(10)* MSA with decreasing re-initialization (MSADR) procedure was used to reduce runtime. In step 4 of SLIE, as in MSA, flow pattern of the next iteration is found by:

$$x_k^{N+1} = x_k^N + \frac{(x_k^{*N} - x_k^N)}{N} \quad (15)$$

where x_k is the flow on link k, and N is the current iteration number. With decreasing re-initialization, the count of N is restarted at one after a pre-fixed number of iterations. The problem is then re-initialized at a decreasing frequency. Convergence is achieved when

$$\max(x_k^{N+1} - x_k^N) \leq e \quad (16)$$

where ε is the convergence parameter. $\varepsilon = 1.0$ was used in this study

MSADR allows the problem to converge with far fewer iterations. In BOWOC, which used the 2020 network and demand set, it took 721 iterations to converge with the SLIE algorithm. With the 2030 inputs, 30-45 iterations were needed to converge, with 21 minutes per iteration on a 3.0GHZ Pentium 4 processor with 1GB RAM. This is much longer than what Bar-Gera and Boyce *(11)* were able to achieve with their non-convex combined model MSA with constant step size implemented on the Chicago Regional Network of nearly 40,000 links using a 2GHz Xeon processor, but much shorter than the straight MSA, which was stopped before convergence could be reached.

Although the program does not integrate directly with existing transportation planning software, data from the standard mode can be post-processed to input into this program and the program can be used on any personal computer to produce results within a reasonable time.

Chapter 4: IMPLEMENTATION FOR TWIN CITIES NETWORK

The purpose of this study was to explore potential efficiencies in the reduction of SOV automobile usage during peak times. Having posed the problem mathematically in the previous chapter, a solution can be implemented for the Twin Cities network using code previously developed for the BOWOC study and the network and demand data supplied by the Twin Cities Metropolitan Council. The BOWOC Fortran code had been tested on the 184-link Waseca test network and produced results identical to those produced by the MINOS software. *(3)* After incorporating the changes discussed, the revised code produced results comparable to the Metropolitan Council's assignment for the 2030 forecast. The revised Fortran code can be found in appendix B.

The most recent Twin Cities network and demand forecasts, for 2030, were obtained from the Metropolitan Council and formatted as needed. Excerpts and descriptions of sample input files are given in appendix C. The original Met Council network file contained 24,308 links, with 3,150 links that led outside of the metropolitan area for which demand was forecasted. Since these "dead links" were not used for any trips under consideration, they were removed to decrease computation time. The resulting network model, representing the seven-county metropolitan area surrounding Minneapolis and St. Paul, contained 12,598 nodes and 21,158 links, of which 1,561 links represented the limited access roadways. The morning peak hour network contained 9 fewer links, due to slight differences in the network caused by some reversible lanes. Note that while it was the limited access roadway system that was of interest as a set for capacity expansion, all roadway links contained in the seven county portion of the model were available for traffic assignment, not just the limited access highways.

One modification to the demand file was necessary to achieve convergence. The trip demand between each O-D pair in the 2030 forecast was given to the hundredths of trips. The file therefore included 1,185,758 O-D pairs with less than one trip demanded, which hindered convergence of the solution procedure. To preserve the total number of trips demanded, the trips between O-D pairs with demand less than one (amounting to 19% of the total demand) were reallocated proportionally to O-D pairs with demand greater than or equal to one. This decreased the number of O-D pairs from 1,372,972 to 187,214, reducing the runtime of each iteration as well as the number of iterations necessary for convergence.

As stated above, with these files, each iteration needed approximately 21 minutes, with 30 to 45 iterations needed for convergence. This runtime was sufficient to allow for solution of several series of scenarios as well as solving for two levels of service. The lower thresholds for both LOS C and LOS D were solved to accommodate differing views on an acceptable level of service. Therefore, for the LOS C solution, any capacity expansion less than the solution level will result in worse than LOS C on at least one link. Similarly, for the LOS D solution, anything less will result in LOS E or F on at least one link. According to the Highway Capacity Manual (HCM) 2000 *(12)*, "LOS C provides for flow with speeds at or near the FFS of the freeway. Freedom to maneuver within the traffic stream is noticeably restricted, and lane changes require more care and vigilance on the part of the driver." One step below that, "LOS D is the level at which speeds begin to decline slightly with increasing flows and density begins to increase somewhat more quickly. Freedom to maneuver within the traffic stream is more noticeably limited, and the driver experiences reduced physical and psychological comfort levels." It is assumed that once speeds are affected, a driver will define that as congestion. However, LOS C is difficult to achieve in an urban area with limited funds and rights of way. LOS D may be tolerable to drivers, but less than that and "maneuverability within the traffic stream is extremely limited, and the level of physical and psychological comfort afforded the driver is poor." While "at its highest density value, LOS E describes operation at capacity … any disruption of the traffic stream, such as vehicles entering from a ramp or a vehicle changing lanes, can establish a disruption wave that propagates throughout the upstream traffic flow. At capacity, the traffic stream has no ability to dissipate even the most minor disruption, and any incident can be expected to produce a serious breakdown with extensive queuing." Even though LOS E accommodates the greatest travel demand, it is difficult to sustain and uncomfortable for drivers. Thus, results were limited to achieving at least LOS D. Solving for both LOS C and D allows for sensitivity analysis of LOS chosen and examination of the tradeoff between driver comfort and additional capacity needed. For free-flow speeds of 60 mph, the volume to capacity ratio cutoffs for LOS C and LOS D are 0.68 and 0.88, respectively.

Results

To examine the scope of the congestion issue, traffic assignment was carried out for a no build scenario, where all travel is by single-occupant vehicles (SOV). If no additions were made to the network, 86% of the roads would be operating at worse than LOS C during the afternoon peak and 73% at worse than LOS D. Even in the morning, 58% of the limited access roads would perform at worse than LOS C during peak hour and 45% would provide less than LOS D.

Next, a roads-only minimum expansion solution was found, where again all travel is by SOV. If all of that congestion were improved to no worse than LOS C on each of the freeway links, an additional 2,619 lane-miles (4,215 lane-km) would be required to accommodate both the morning and afternoon peak hours, more than twice the 2,588 existing lane-miles (4,165 lane-km). Allowing up to LOS D would require 1,578 lane-miles (2,540 lane-km). FIGURE 4.1 maps the lanes to be added.

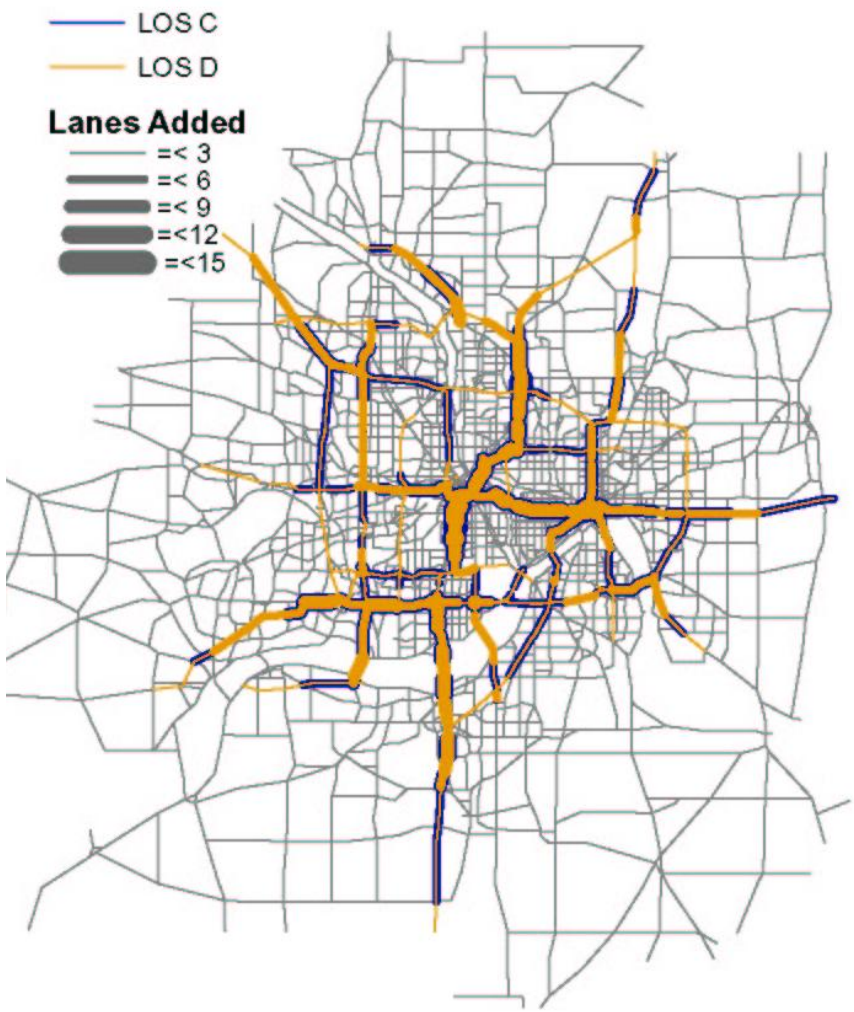

FIGURE 4.1 Capacity expansion needed to achieve LOS C and D on Twin Cities limited access highways during both morning and afternoon peak hours (taking maximum for each direction in AM and PM solution).

If demand on the roads were decreased by 20%, required lane-miles would decrease to 1,689 (2,718 lane-km) for LOS C and 906 (1,458 lane-km) for LOS D. These are 36% and 43% decreases in capacity expansion, respectively, compared to the expansion required for the full SOV demand scenario. A 10% decrease in demand yields 18% and 22% decreases in required capacity expansion. For LOS D, a given percentage decrease in demand results in a decrease in required capacity expansion of more than twice that percentage. (TABLE 4.2)

If, however, 20% of trips demanded are made by transit, and those transit trips are loaded onto standard buses directly from origin to destination, for LOS C, only 82 lane-miles, or 3%, of capacity expansion would be saved. For LOS D, 20% transit share yields only a 62 lane-mile savings, only 4% less than the expansion necessary to accommodate all travel by SOVs. This ineffectiveness is due to the low occupancy of the transit vehicles that are included in the system. As shown in TABLE 4.1, with vehicles dedicated to each O-D pair individually, even at 20% transit share, 98.9% of the O-D pairs had fewer than 10 transit trips each. 91.5% had fewer 2.5 transit trips and thus did not warrant having transit vehicles at all. In total, this was 51.3% of the 20% of demand that was designated for transit. However, because of their spatial distribution, these transit trips could not be accommodated and were re-assigned to SOV. This reduced total transit share to less than 20% of the total travel demand, but this seemed more reasonable than actually *increasing* trips through the use of transit on these routes. Only 0.3% of the O-D pairs had transit vehicle occupancies of more than 25, half of each vehicle's maximum occupancy. Considering the inefficiency of this formulation it is unlikely for any real transit system to produce returns like these. It is interesting to note, though, that even with a highly inefficient transit system, some gains can be made in decreasing roadway expansion.

TABLE 4.1 Distribution of Transit Trips, Assuming Uniform 20% Transit Share Across O-D Matrix

Number of Transit Trips per O-D Pair	Number of Transit Vehicles per O-D Pair	Number of O-D Pairs	Percentage of O-D Pairs	Total Number of Transit Trips in Group	Percentage of Total Transit Trips
< 2.5	0	171349	91.5	115431.5	51.3
2.5-10	1	13838	7.4	61071.3	27.1
10-25	1	1570	0.8	22622.6	10.1
25-50	1	320	0.2	10669.7	4.7
>50	>1	137	0.1	15251.3	6.8

A more realistic system aggregating transit trips would lie between these two bounds. Using the TAD aggregation, the results more closely follow the results with no transit vehicles added back in, as shown in TABLE 4.2 and FIGURE 4.2. With transit trips made on standard buses between TADs, a 20% transit share reduces capacity expansion by 30% (774 lane-miles or 1246 lane-km) for LOS C and 36% (565 lane-miles or 909 lane-km) for LOS D.

TABLE 4.2 Percentage Reduction in Required Capacity Expansion

Percent Transit	No Transit Vehicle Interaction, LOS D	No Transit Vehicle Interaction, LOS C	2.5 PCE, TAD, LOS D	2.5 PCE, TAD, LOS C	2.5 PCE, TAZ, LOS D	2.5 PCE, TAZ, LOS C
1	2.2	1.8	0.9	0.7	0.0	0.0
2	4.5	3.6	2.1	1.7	0.0	0.0
5	11.0	9.0	7.1	5.8	0.1	0.1
10	21.9	17.9	16.6	13.5	0.8	0.6
15	32.4	26.8	26.3	21.5	2.1	1.6
20	42.6	35.5	35.8	29.5	3.9	3.1

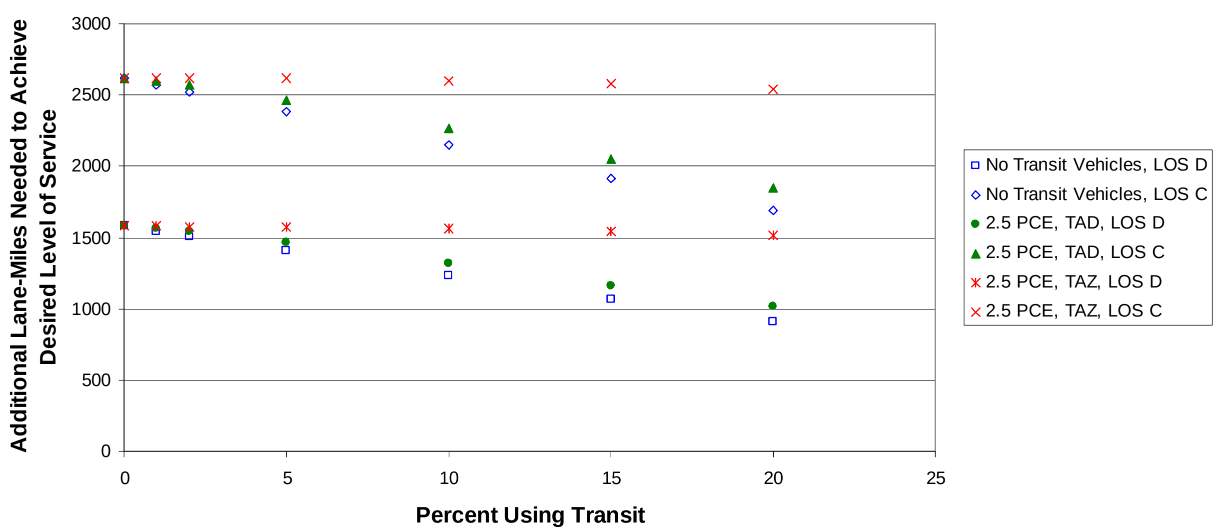

FIGURE 4.2 Capacity expansion necessary to achieve LOS C and D with the three different transit formulations.

As shown in FIGURE 4.2, the choice of service level turned out to make a significant difference in capacity expansion necessary. With no demand reduction, to provide LOS C required 1,041, or 66%, more lane-miles (1,675 lane-km) compared to LOS D. With 20% demand reduction, LOS C required 67% to 86% more lane-miles over LOS D for the three different scenarios. However, each level of service showed similar sensitivity to the different transit formulations.

Chapter 5: SOLUTION POST PROCESSING - EXPLORING COST FORMULATIONS

With a NDP solution there are various post-processing calculations that could be done, one of which is calculating how much it would take to build the number of lane-miles arrived at to satisfy future travel demand in uncongested conditions. This was explored as part of the previous work, BOWOC, but the procedures explored at the time had drawbacks that warranted not using them. Cost was a frequent question, though, about the results, and so deserved further investigation. For BOWOC, the optimization function was based on minimizing the number of lane-miles to be constructed, which assumes that costs are the same throughout the metropolitan area. While not an unreasonable assumption in the absence of a good cost model, minimizing costs rather than lane-miles would be a more realistic representation of the decision-making process. Thus, cost estimation formulations were reviewed. There are basically three methods for construction cost estimation:

1. Unit rates of construction (dollars per mile by highway type)

Individual site conditions such as topography, in situ soil, land prices, environment, and traffic loads vary greatly from location to location to make average prices inaccurate estimates of the price of individual projects or apparently, even of all projects in a particular year.

2. Extrapolation of past trends

These are used to forecast future overall construction rates, and include examples such as time series analysis. Typically these are collapsed into a single overall expression of construction cost such as Engineering News Record's Construction Cost Index (ENR CCI). An issue with these models is that they rely on the notion that past conditions will be maintained in the future. They are not useful for optimization type problems.

3. Construction cost models based on factors believed to influence construction costs

The relationship between construction costs and these factors is based on past records of construction costs. Factors that seem to have a significant impact on contract costs include season, location, type of project, contract duration and contract size.

Literature Review

Below is a discussion of the different construction cost forms that were reviewed.

A. Abdulaal and LeBlanc *(13)* explored a number of construction cost formulations, including a quadratic form in order to examine how the solution responds under different cost formulations, such as concave versus convex. Abdulaal and Leblanc discuss that in continuous network design problems that convex investment costs usually results in minor increases in practical capacity for many arcs proposed for improvements, which is useful if the purpose is to maintain/improve an existing network. Concave functions are more appropriate if the purpose is to build new roads, as large increases of only some of the proposed arcs results. An interesting point that Abdulaal and LeBlanc make is that they allow for investment cost functions to be defined for negative values of y_k - although negative values of y_k have no physical meaning, allowing them to be unrestricted allows the computation to be unconstrained. Intuitively, the optimal capacity expansion cannot be negative because a positive solution would be better. While the impact of these different forms is discussed for the example cost formulation, these cost formulations have no realistic basis.

B. Keeler & Small *(14)* formulated a construction cost per lane-mile, given by:

$$KLM = \exp(a_1 CRS + a_2 CUC + a_3 FR + a_4 FSU + a_5 FC) w^{a6} \qquad (17)$$

where

CRS = non-freeway rural roads $\quad$ CUC = urban roads

FR = rural freeways $\quad$ FSU = suburban freeways

FC = urban freeways $\quad$ w = average roadway width

a_i = estimated parameters

C. Small et al. *(15)* linearized the Keeler and Small earlier work to give a construction cost per mile:

$$K(W,D) = \begin{cases} 0 & \textit{if } W = 0 \\ k_0 + (k_1 + k_2 D)W & \textit{if } W > 0 \end{cases} \qquad (18)$$

where

W = roadway width (no. of lanes)

D = pavement thickness

k_i = coefficients

Substituting suggested values, construction cost then is:

$$K(W,L) = \begin{cases} 0 & \textit{if } W = 0 \\ 95{,}000 + (950{,}000)WL & \textit{if } W > 0 \end{cases} \qquad (19)$$

where

L = roadway length

This is a discrete equation, whereas continuous functions are more useful for optimization work, so a continuous version of this was formulated. Assuming one lane is equivalent to a volume of 1200 vehicles per hour then:

$$g_k(y_k) = \begin{cases} 0.07(y_k^2) + 792(y_k) & \textit{if } y_k < \textit{1200} \\ 95{,}000 + (950{,}000/1200)(y_k l_k) & \textit{if } y_k \geq \textit{1200} \end{cases} \quad (20)$$

D. Levinson & Ramachandra *(16)* cost model:

$$E_{ij} = f(L_{ij} * \Delta C_{ij}, F, N, T, Y, D, X) \quad (21)$$

where

$L_{ij} * \Delta C_{ij}$ = lane-miles of construction

F = funding program dummy variable

N = new construction or expansion dummy variable

T = interstate or state highway program dummy variable

Y = year of completion minus 1979

D = duration of construction

X = distance of link from nearest downtown

A simplified version of this, substituting all the constants gives the following formulation:

$$\text{Construction Cost (\$ `000s)} = (\text{lane-miles})^{b1}(\text{year})^{b2} e^{b3(\text{type})+b4} \quad (22)$$

where

b1 = lane-miles of roadway

b2 = forecast construction year minus the base year (1979)

b3 = 1 if a limited access road, 0 otherwise

b4 = constant

While this formulation is very attractive because it was developed based on Minnesota data and therefore is the only equation with a realistic basis for developing cost estimates, it turns out this form is very sensitive to how the expansion is broken into separate projects, and how these are sequenced in time. That is, if you plug in 1143 lane-miles, and a 40-year time horizon, you get a cost of about $ 430 million, but if you treat the expansion as 1143 separate 1 lane-mile projects, you get a cost of about $18.4 billion. If you use a 20-year time horizon, the costs are about half of these.

A construction cost formulation for planning level estimates would be an extremely useful tool not only for this type of project, but for many planning projects as well. None of the formulations investigated here however are suitable for the purposes of this project. Ideally the equation would require a limited amount of data, such as lane miles of roadway, construction year and roadway type, and would be a continuous function for use in optimization work.

Chapter 6: CONCLUSIONS AND FUTURE RESEARCH

This study explores the relationship between reducing peak automobile demand in order to satisfy a particular level of service on the limited access roads. The level of service chosen affected the amount of expansion necessary, but not the relationship between transit share and capacity expansion within each transit formulation. Effectiveness varied widely with different formulations of transit, from 4% to 43% reductions in capacity expansion at LOS D with 20% transit share. Reducing auto demand, or increasing alternate transportation mode usage such as transit, could help reduce capacity expansion needed for LOS D by 36% if 20% of trips were reduced – equivalent to one day per week of alternate travel. Even though 64% of the capacity expansion is still 1,013 lane-miles (1,630 lane-km) needed to achieve LOS D, it is still a little less congestion the other four days of the week even without any expansion.

While 20% transit share may seem an unrealistic increase over current transit usage in the Twin Cities area, it is low for many cities around the world. In the last year alone, gas price increases have had gotten more people to consider alternate modes and fuel-efficient vehicles. There is no indication that gas prices should stay stable over the next 20 years and as yet no guarantee of alternative power sources for personal vehicles. More controllably, cities have successfully used pricing and land use strategies to encourage transit use in long-range planning. Cities can also adopt more immediate demand-side changes, such as requiring each commuter to use transit or telecommute at least one time per week.

If there are no major disturbances and gas prices increase only with inflation and local planning principles remain the same, then transit might continue to account for only around 5% of commuting trips. Without considerable expansion of transit, it is still possible to decrease capacity expansion for LOS D by 7.1%, or 113 lane-miles (182 lane-km), with an express bus type transit system. If 5% of trips are completely removed from the system, the decrease goes up to 11%, or 175 lane-miles (282 lane-km).

For more accurate estimates of the effects of an express bus system, aggregation of the transit trips might be based more on the distribution of trip origins and destinations rather than the TADs which are based on political boundaries. More work can also be done to represent transit in a way that more accurately reflects the real transit system by assigning transit in proportions similar to the current transit use, rather than uniformly across all O-D pairs. Further research could also be done to verify that multimodal trips would not significantly change the results. Trips to the park-and-ride are likely to be made by personal vehicle, but they are unlikely to use limited access highways.

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Appendix A: Verification of Algorithm Validity

Recall our variant of the continuous network design problem, written in vector notation:

NDP*

Minimize:

$$\vec{l}^{\,T}\vec{y} = \sum_{i=1}^{m} l_i y_i$$

subject to:

$$-\tilde{C}\vec{y} + \vec{x}^1 - \tilde{C}\vec{z} \le \vec{0} \qquad \text{i.e. } \frac{x_i}{z_i + y_i} \le \tilde{c}_i \qquad i = 1,\ldots,m$$

$$-\vec{y} \le 0 \qquad \text{i.e. } y_i \ge 0 \qquad i = 1,\ldots,m$$

$$\vec{h}(\vec{x},\vec{y}) = \vec{0} \qquad \text{i.e. } h_k(\vec{x},\vec{y}) = x_k - \sum_j d_j q_{jk}(\vec{x},\vec{y}) = 0 \quad k = 1,..,n$$

where

$\tilde{C} = \text{diag}\{\tilde{c}_i\}$, the links which are candidates for expansion are indexed i=1,…,m,

$\vec{x}^1 = [x_1,..,x_m]^T$ is the vector whose elements are the flows on the set of candidate links,

$q_{jk}(\vec{x},\vec{y})$ denote the logit model's proportion of trips between OD pair j using link k.

We want to show:

The tuple $\bar{x},\bar{y}$ is a Karush-Kuhn-Tucker (KKT) point for NDP*, if and only if $\bar{x},\bar{y}$ satisfies the equations

$$\bar{y}_i = \max(0, \frac{\bar{x}_i}{\tilde{c}_i} - z_i), \quad i = 1,..,m$$

$$\bar{x}_k - \sum_j d_j q_{jk}(\bar{x},\bar{y}) = 0, \quad k = 1,..,n$$

To do this, we will use Theorem 4.2.15 in Bazaraa, Sherali, and Shetty *(17)*, which established an equivalence between a KKT point of a nonlinear program and the solution to a linear programming approximation. We should point out that this theorem does not claim to provide an algorithm for solving NDP*, it only provides an alternative way of characterizing the problem's KKT points.

First, suppose $\bar{x}, \bar{y}$ is a KKT point for NDP*. Then, by Theorem 4.2.15 in Bazaraa, Sherali and Shetty, this is true if and only if $\bar{x}, \bar{y}$ is also a solution to the approximating linear program

LP*

Minimize:

$$\vec{l}^{T}\bar{y} + \vec{l}^{T}\left(\vec{y} - \bar{y}\right)$$

subject to:

$$-\tilde{C}\bar{y} + \bar{x}^{1} - \tilde{C}\vec{z} - \tilde{C}[\vec{y} - \bar{y}] + [\vec{x}^{1} - \bar{x}^{1}] \leq \vec{0}$$

$$-\mathrm{I}\bar{y} - \mathrm{I}\left(\vec{y} - \bar{y}\right) \leq 0$$

$$\vec{h}(\bar{x}, \bar{y}) + H_{1}(\bar{x}, \bar{y})[\vec{y} - \bar{y}] + H_{2}(\bar{x}, \bar{y})[\vec{x} - \bar{x}] = \vec{0}$$

where H_1 and H_2 denote matrices of partial derivatives of the equality constraints. Since $\bar{x}, \bar{y}$ is feasible $\vec{h}(\bar{x}, \bar{y}) = \vec{0}$, the approximating equality constraints imply

$$[\vec{x} - \bar{x}] = -H_{2}^{-1}(\bar{x}, \bar{y})H_{1}(\bar{x}, \bar{y})[\vec{y} - \bar{y}]$$

and the existence of the inverse matrix H_2^{-1} follows from the standard conditions guaranteeing the existence and uniqueness of the stochastic user equilibrium assignment (Daganzo *(18)*, Sheffi *(6)*, Davis *(19)*, p. 102-104). More particularly, letting J denote the first m rows of $-H_2^{-1}H_1$, we have $[\vec{x}^{1} - \bar{x}^{1}] = J(\bar{x}, \bar{y})[\vec{y} - \bar{y}]$, and LP* can be simplified to

minimize:

$$l^{T}\vec{y}$$

subject to:

$$[J(\bar{x}, \bar{y}) - \tilde{C}]\vec{y} \leq \tilde{C}\vec{z} + J(\bar{x}, \bar{y})\bar{y} - \vec{x}^{1} = \vec{b}$$

$\vec{y} \geq \vec{0}$

Introducing the slack variable $\vec{w}$, we can then write this in a "standard" form

Minimize:

$$l^T \vec{y}$$

subject to:

$$[J(\bar{x}, \bar{y}) - \tilde{C}]\vec{y} + I\vec{w} = \tilde{C}\vec{z} - J(\bar{x}, \bar{y})\bar{y} - \vec{x}^1$$

$$\vec{y} \geq \vec{0}$$
$$\vec{w} \geq \vec{0}$$

The extreme points of the polyhedral region:

$$[J(\bar{x}, \bar{y}) - \tilde{C} \mid I]\begin{bmatrix} \vec{y} \\ \vec{w} \end{bmatrix} = \vec{b}$$
$$\vec{y} \geq 0$$
$$\vec{w} \geq 0$$

can be expressed, by re-arranging the order of the elements of $\vec{y}$ and $\vec{w}$, as

$$\begin{bmatrix} \vec{y}_1 \\ \vec{0} \\ \vec{0} \\ \vec{w}_2 \end{bmatrix} \text{ with } \begin{matrix} \vec{y}_1 \geq \vec{0} \\ \vec{w}_2 \geq \vec{0} \end{matrix}$$

In particular, since $\bar{x}, \bar{y}$ solves LP* there is a corresponding extreme point:

$$\begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \\ \bar{w}_1 \\ \bar{w}_2 \end{bmatrix} \text{ with } \bar{y}_2 = \vec{0}, \bar{w}_1 = \vec{0}$$

which satisfies $\bar{y}_1 \geq \vec{0}, \bar{w}_2 \geq \vec{0}$ and

$$
\begin{bmatrix} J_1(\bar{x},\bar{y}) - \tilde{C}_1 & J_2(\bar{x},\bar{y}) & I & 0 \\ J_3(\bar{x},\bar{y}) & J_4(\bar{x},\bar{y}) - \tilde{C}_2 & 0 & I \end{bmatrix} \begin{bmatrix} \bar{y}_1 \\ \vec{0} \\ \vec{0} \\ \bar{w}_2 \end{bmatrix} =
$$

$$
\begin{bmatrix} \tilde{C}_1 & 0 \\ 0 & \tilde{C}_2 \end{bmatrix} \begin{bmatrix} \vec{z}_1 \\ \vec{z}_2 \end{bmatrix} + \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \bar{y}_1 \\ \vec{0} \end{bmatrix} - \begin{bmatrix} \bar{x}_1^1 \\ \bar{x}_2^1 \end{bmatrix}
$$

This simplifies to

$$
\begin{aligned}
&-C_1\bar{y}_1 = \tilde{C}_1\vec{z}_1 - \bar{x}_1^1, \quad \bar{y}_1 \geq \vec{0} \\
&\bar{w}_2 = \tilde{C}_2\vec{z}_2 - \bar{x}_2^1, \quad \bar{w}_2 \geq \vec{0}
\end{aligned}
$$

The first of these two equations implies

$$
\begin{aligned}
\bar{y}_1 &= -\tilde{C}_1^{-1}\left(\tilde{C}_1\vec{z}_1 - \bar{x}_1^1\right) \\
&= \tilde{C}_1^{-1}\bar{x}_1^1 - \vec{z}_1
\end{aligned}
$$

and since $\bar{y}_2 = \vec{0}$ we have shown that $\bar{x}, \bar{y}$ satisfies the equations

$$
\bar{x}_k = \sum_j d_j q_{jk}(\bar{x},\bar{y}) \quad k = 1,..,m
$$

$$
\bar{y}_i = \max\left(0, \frac{\bar{x}_i}{\tilde{c}_i} - z_i\right) \quad i = 1,..,m
$$

On the other hand, suppose $\bar{x}, \bar{y}$ satisfies the above equations, but is not a KKT point for NDP*. Then, constructing the linear programming approximation LP* using first-order Taylor expansions around $\bar{x}, \bar{y}$, Bazaraa, Sherali and Shetty's theorem implies that $\bar{x}, \bar{y}$ also does not solve the approximating problem LP*. But $\bar{x}, \bar{y}$ is feasible for LP*, so there must exist a vector $\vec{v}$ such that $\bar{y} + \vec{v}$ is also feasible and $\vec{l}^T[\bar{y} + \vec{v}] < \vec{l}^T\bar{y}$. Since $l_i > 0$ for i=1,..,m, this implies that there exists at least one element of $\vec{v}$, such that $v_a < 0$. But then if $\bar{y}_a = 0$, $\bar{y}_a + v_a < 0$, while if $\bar{y}_a = \bar{x}_a / \tilde{c}_a - z_a$ then $\bar{x}_a /(z_a + \bar{y}_a + v_a) > \tilde{c}_a$, contradicting the assumption that $\bar{y} + \vec{v}$ is feasible. So $\bar{x}, \bar{y}$ must solve LP*, and hence provide a KKT point for NDP*.

Next, we will show that our iterative scheme converges to a KKT point for NDP*, using a straightforward adaptation of the proof method used by Powell and Sheffi *(20)*. To restate, our iterative scheme takes the form

$$y_i^{N+1} = \begin{cases} 0, & x_i^N \le \tilde{c}_i z_i \\ \dfrac{x_i^N}{\tilde{c}_i} - z_i & x_i^N > \tilde{c}_i z_i \end{cases}$$

$$x_k^{N+1} = x_k^N + \left(\frac{1}{N}\right)\left[\sum_j d_j \left(\sum_r d_{jrk}\, p_{jr}(\mathbf{x}^N, \mathbf{y}^{N+1}\right) - x_k^N\right]$$

As did Powell and Sheffi, let's first consider an iterative scheme on the path flows

$$d_{jr}^{N+1} = d_{jr}^N + \left(\frac{1}{N}\right)\left(d_j p_{jr}(\mathbf{x}^N, \mathbf{y}^{N+1}) - d_{jr}^N\right)$$

Since the link flows x_k are well-defined functions of the path flows, and the link capacity expansions y_i are well-defined functions of the link flows, this iteration is in turn well-defined, and we need to show that this iterative scheme satisfies Powell and Sheffi's conditions (2.2a-2.2d). It is well known that the updating weights (1/N) satisfy (2.2a), and to show that the other conditions are also satisfied consider the modification of Fisk's *(21)* optimization criterion

$$f(\mathbf{d}) = \left(\frac{1}{q}\right)\sum_j \sum_r d_r^j \log(d_r^r) + \sum_k \int_0^{x_k} c_k(w, y_k(w))dw$$

where the capacity expansion functions $y_k(x)$ are defined via

$$y_i = \begin{cases} 0, & x_i \le \tilde{c}_i z_i \\ (x_i / \tilde{c}_i) - z_i & x_i > \tilde{c}_i z_i \end{cases} \quad i = 1,..,m$$

$$y_k = 0, \quad k = m+1,..,n$$

This function is undefined whenever a route flow $d_{jr}=0$, but the first order conditions for optimality imply a set of route flows that minimizes f satisfy

$$d_{jr} \propto \exp(-qc_{jr}) > 0$$

so it can be shown that at least in a neighborhood of an minimizing solution *f* is strictly convex and so condition (2.2b) is satisfied. Next, since

$$\sum_j \sum_r \left(\frac{\partial f}{\partial d_{jr}} \right) \left(d_j \frac{\exp(-qc_{jr})}{\sum_s \exp(-qc_{js})} - d_{jr} \right) \le 0$$

and equals 0 only when $d_{jr}=d_j p_{jr}$, *f* is a descent function for our route flow iteration, and (2.2c) is satisfied. Finally, the boundedness of *f* and its derivatives can be established using an argument similar to that in Powell and Sheffi, and so it follows that the sequence of updated route flows d_{jr}^N converges to a solution to the system of equations

$$d_{jr} = d_j p_{jr}(\vec{x}, \vec{y}(\vec{x}))$$

Since the link flows and capacity expansions are continuous functions of the route flows, the sequence x_k^N, $y_i(x_i^N)$ converges to

$$\bar{x}_k = \sum_j d_j q_{jk}(\bar{x}, \bar{y}) \quad k = 1,.., n$$

$$\bar{y}_i = \max\left(0, \frac{\bar{x}_i}{\tilde{c}_i} - z_i \right) \quad i = 1,.., m$$

which characterize the KKT points for NDP*.

Appendix B: Computer Code

```
*********************************************************************
****                    Start of MAIN program                    ****
*********************************************************************

      PROGRAM            MAIN
      IMPLICIT           DOUBLE PRECISION (A-H,O-Z)

      INTEGER LIMIT
      PARAMETER (LIMIT=25000)
           LOGICAL NPOSSLST(LIMIT)

      DOUBLE PRECISION YGUESS(limit)

      COMMON /KSINPUT1/ NNODE, NUMORG,
     & NEXT(15000), NPRED(15000),
     & NTONODE(25000),NFMNODE(25000),XGUESS(25000),
     & GLINK(25000,2),SLINK(25000,4), CLINK(25000,6),NIPE(15000)

        COMMON /KSINPUT2/ ABI(15000),
     & D(2000,15000),NLINK, NPOINT(15000)
      COMMON /KSINPUT2a/ THETA
        COMMON /KSINPUT3/ IDERIV,  NPOSS, NPOSSLST
        COMMON /KSINPUT4/ NUMB, NAME

C  SUBROUTINE MOOREPAPE READS IN DATA, SETS VARIABLES AND SETS UP
C  INITIAL FEASIBLE SOLUTION TO START OPTIMIZER.

      CALL MOOREPAPE

c     Read flows from file named 'input'.  If file named 'input'
c     can't be opened, just start with empty network.

      OPEN (UNIT=50,FILE='input',STATUS='OLD',ERR=12)
      READ(50,*) ITNUM, ERRMAX
      do 11 k=1,NLINK
         read(50,*) xguess(k), YGUESS(k)
 11   CONTINUE
      GOTO 14
 12   DO 13 k=1,NLINK
         xguess(k)=0.0d+0
         yguess(k)=0.0d+0
 13   CONTINUE

 14   CALL MSA(YGUESS)

*     ------------------------------------------------------------------
      stop
      end
```

```
**********************************************************************
****                   Start of subroutine MOOREPAPE              ****
**********************************************************************

      SUBROUTINE  MOOREPAPE

      IMPLICIT DOUBLE PRECISION (A-H,O-Z)

      INTEGER LIMIT,  LIM3
      PARAMETER (LIMIT=25000,LIM3=15000)

      LOGICAL NPOSSLST(LIMIT)
      INTEGER IT(LIM3), LISTY(LIMIT), DEMAND, LKS(10),
     & TYPES, TVREM, NUMTAZ, TAD(1500), REPTAZ(1500),
     & JTRAN, KTRAN

      CHARACTER NAME*20

      DOUBLEPRECISION XT(LIMIT), ZERO, DIFF,
     & ERRMAX, ERRCRIT, DA(LIMIT), FIT, LOS,
     & MDSPLT, TRANDEM, TRANPCE, PCE, SOVDEM, TRANVEH, TVOCC,
     & TD(1500,1500), SD(1500,1500)

      COMMON /KSINPUT1/ NNODE, NUMORG,
     & NEXT(15000), NPRED(15000),
     & NTONODE(25000),NFMNODE(25000),XGUESS(25000),
     & GLINK(25000,2),SLINK(25000,4), CLINK(25000,6),NIPE(15000)

           COMMON /KSINPUT2/ ABI(15000),
     & D(2000,15000),NLINK, NPOINT(15000)
      COMMON /KSINPUT2a/ THETA
        COMMON /KSINPUT3/ IDERIV,  NPOSS, NPOSSLST
        COMMON /KSINPUT4/ NUMB, NAME

      OPEN (UNIT=1,FILE='OD',STATUS='OLD')
      OPEN (UNIT=2,FILE='NET',STATUS='OLD')
      OPEN (UNIT=3,FILE='OUT',STATUS='UNKNOWN')

C *** THETA IS THE PERCEPTION OF THE COST DIFFERENCE
C ***  OF DIFFERENT PATHS.
           THETA = 2.0D-01
           ZERO    = 0.0D+0

C *** READ IN TRANSIT PARAMETERS FROM FILE 'TRANPRM'.  FIRST LINE
C *** IS THE MAXIMUM OCCUPANCY OF THE TRANSIT VEHICLES.  SECOND LINE
C *** IS PASSENGER CAR EQUIVALENT FACTOR FOR THE TRANSIT VEHICLES.
C *** THIRD LINE IS THE MODE SPLIT, EXPRESSED AS THE PERCENTAGE OF
C *** TRIPS MADE IN SINGLE-OCCUPANCY VEHICLES.
C *** IF FILE OPEN FAILS, USE HARD-CODED DEFAULTS.

      OPEN (UNIT=4,FILE='TRANPRM',STATUS='OLD',ERR=60)
      READ (4,*) TVOCC
      READ (4,*) PCE
      READ (4,*) MDSPLT
      GOTO 61
```

```
   60  TVOCC = 50
       PCE = 2.5D+0
       MDSPLT = 1.0

 c *** READ IN TAD FILE IF AGGREGATING TRANSPORTATION ANALYSIS ZONES (TAZS)
 C     INTO DISTRICTS (TADS).  IF NO TAD FILE, SKIPS TO COMMAND 70 TO DO
 C     SAME PROCEDURE FOR FIGURING TRANSIT VEHICLES WITHOUT COMPLICATION OF
 C     TADS.
 C *** 'NUMTAZ' IS THE NUMBER OF TAZS IN THE NETWORK.
 C *** 'TAD(M)' IS THE DISTRICT TO WHICH ZONE M BELONGS
 C *** 'REPTAZ(M)' IS THE REPRESENTATIVE TAZ, IE, THE ZONE TO WHICH ALL
 C        TRANSIT VEHICLES WITHIN THE DISTRICT WILL BE ASSIGNED.

   61  open (unit=5,file='tad',status='OLD',ERR=70)
       read (5,*) numtaz
        do 62 i=1,numtaz
           read (5,*) m, tad(m), reptaz(m)
   62   continue


 C *** READ IN NON-ZERO DEMAND VALUES,
 C *** 'DEMAND' IS THE NO. OF PEOPLE WANTING TO TRAVEL FROM A->B.
 C *** MDSPLT IS THE MODAL SPLIT, AS PERCENTAGE OF CARS.
 C *** CALCULATE TRANSIT DEMAND, ASSIGN TRANSIT DEMAND TO PROPER
 C      REPRESENTATIVE ZONE ('REPTAZ').
 C     CALCULATE NUMBER OF TRANSIT VEHICLES NEEDED, AND ADD THE PCE
 C      OF THE TRANSIT VEHICLES BACK INTO DEMAND FOR THAT ZONE.

       READ (1,*) DEMAND
         DO 40 I=1,DEMAND
            READ(1,*) J,K,D(J,K)
            IF (MDSPLT.NE.1) THEN
              SOVDEM=MDSPLT*D(J,K)
              TRANDEM=(1.0D+0-MDSPLT)*D(J,K)

                DO 63 M=1, NUMTAZ
                  IF (M.EQ.J) JTRAN=REPTAZ(M)
                  IF (M.EQ.K) KTRAN=REPTAZ(M)
   63           CONTINUE
                TD(JTRAN,KTRAN)=TD(JTRAN,KTRAN)+TRANDEM
                SD(J,K)=SOVDEM
          ELSE
            TD(J,K)=ZERO
            SD(J,K)=D(J,K)
            END IF
   40    CONTINUE
         DO 64 J=1,NUMTAZ
           DO 65 K=1,NUMTAZ
             TRANVEH=TD(J,K)/TVOCC
             TVREM=MOD(TD(J,K),TVOCC)
             IF (TVREM.GE.1.0D+0) THEN
               TRANVEH=NINT((TD(J,K)/TVOCC)+5.0D-1)
             ELSE
               TRANVEH=INT(TRANVEH)
             END IF
             TRANPCE=PCE*TRANVEH
```

```
            D(J,K)=SD(J,K)+TRANPCE
   65      CONTINUE
   64    CONTINUE

C *** IF NO TRANSPORTATION ANALYSIS DISTRICTS ARE TO BE USED (NO TAD FILE),
C ***   THE FOLLOWING SECTION (70-71) IS USED
C *** READ IN NON-ZERO DEMAND VALUES,
C *** 'DEMAND' IS THE NUMBER OF OD PAIRS.
C *** CALCULATE TRANSIT DEMAND, NUMBER OF TRANSIT VEHICLES NEEDED,
C ***   AND ADD THE PCE OF THE TRANSIT VEHICLES BACK INTO DEMAND.
C *** IF THE NUMBER OF TRANSIT TRIPS DEMANDED IS LESS THAN THE PCE,
C ***   ASSUME NO TRANSIT AVAILABLE.

   70  READ (1,*) DEMAND
         DO 71 I=1,DEMAND
            READ(1,*) J,K,D(J,K)
            IF (MDSPLT.NE.1) THEN
              SOVDEM=MDSPLT*D(J,K)
              TRANDEM=(1.0D+0-MDSPLT)*D(J,K)
              IF (TRANDEM.LT.PCE) GOTO 71
              TRANVEH=TRANDEM/TVOCC
              TVREM=MOD(TRANDEM,TVOCC)
                IF (TVREM.GE.1.0D+0) THEN
                   TRANVEH=NINT((TRANDEM/TVOCC)+5.0D-1)
                ELSE
                   TRANVEH=INT(TRANVEH)
                END IF
              TRANPCE=PCE*TRANVEH
              D(J,K)=SOVDEM+TRANPCE
            END IF
   71    CONTINUE

C *** INPUT TITLE OF PROBLEM TO BE SOLVED,
C ***   THIS SETS WHICH PARAMETERS ARE TO BE READ.

   80  READ(2,*) NUMB
       READ(2,*) NAME

C *** INPUT NUMBER OF ORIGINS, NUMBER OF NODES, NUMBER OF LINKS,
C ***   ROAD TYPES TO BE EXPANDED, AND LEVEL OF SERVICE DESIRED

     1 READ(2,*) NUMORG,NNODE,NLINK
       READ(2,*) TYPES,(LKS(L),L=1,TYPES)
       READ(2,*) LOS

C *** INPUT OF THE NETWORK
C *** READ START AND END NODES FOR LINKS &
C *** READ IN DATA ASSOC W LINK CHARACTERISTICS.
C        CLINK-1=ffs tvl time (HRS)
C        CLINK-2=0.15 (BPR Coeff)
C        CLINK-3=capacity
C        CLINK-4=BPR tvl time
C        SLINK-1=FFS (mph)
C        SLINK-2='functional class'
C        SLINK-3=area type
C        SLINK-4=#-lanes
```

```
C        GLINK-1=link length (mi)
C        GLINK-2=LOS

      NPOSS=0
      DO 4 I=1,NLINK
C *** FOR PROBLEMS 1 OR 2 DATA INPUT IS SLIGHTLY DIFFERENT
        IF (NUMB.LE.2) THEN
          READ(2,*) NFMNODE(I),NTONODE(I),CLINK(I,3),
     &     GLINK(I,1), SLINK(I,1)
          DO 31 L=1,TYPES
                 IF (SLINK(I,1).EQ.LKS(L)) THEN
                   NPOSS=NPOSS+1
                   LISTY(NPOSS)=I
                   GLINK(I,2) = LOS
                 ENDIF
  31      CONTINUE
        ENDIF
C *** FOR PROBLEMS 3(TC) DATA INPUT IS DIFFERENT
        IF (NUMB.EQ.3) THEN
          READ(2,*) NFMNODE(I),NTONODE(I),
     &      SLINK(I,2), GLINK(I,1),SLINK(I,1),
     &      SLINK(I,3),SLINK(I,4), CLINK(I,3)
          DO 32 L=1,TYPES
                 IF (SLINK(I,2).EQ.LKS(L)) THEN
                   NPOSS=NPOSS+1
                   LISTY(NPOSS)=I
                   GLINK(I,2) = LOS
                 ENDIF
  32      CONTINUE
        ENDIF
C *** USE SLINK(K,2) FOR ASS GRP OR AREA TYPE FOR VC LOS.
        CLINK(I,2)=1.5D-01
          if (CLINK(I,3).EQ.0.0d+0) CLINK(I,3)=2.4d+03
          if (SLINK(I,1).EQ.0.0d+0) SLINK(I,1)=1.5d+01
C *** CONVERT TIME TO MINUTES.
        CLINK(I,1)=6.0D+01*(GLINK(I,1)/SLINK(I,1))
    4 CONTINUE
C IF LINK CAN BE EXPANDED THEN TRUE.
      DO 35 K=1,NPOSS
        NPOSSLST(LISTY(K))=.TRUE.
   35 CONTINUE
C *** FILL NPOINT WITH THE START OF NEW NODE NOS.
      K=0
      J=1
      DO 44 I=1,NLINK

  45    IF (NFMNODE(I).EQ.J) THEN
            K=K+1
            IT(J)=K
          ELSE
            K=0
            J=J+1
            IT(J)=K
            GOTO 45
          ENDIF
  44  CONTINUE
```

```
      J=0
      DO 46 I=1,NNODE
        J=J+IT(I)
        NEXT(I)=J
  46  CONTINUE
c
C ** FILL NPOINT WITH THE START OF NEW NODE NOS.
      NPOINT(1)=1
      DO 47 I=2,NNODE
        NPOINT(I)=NPOINT(I-1)+IT(I-1)
  47  CONTINUE

C       INITIAL DIAL ASSIGNMENT TO DETERMINE EFFICIENT PATHS
           DO 93 K=1,NLINK
        XGUESS(K) = ZERO
             CLINK(K,4)=CLINK(K,1)
  93  CONTINUE

C *** DONE WITH MOOREPAPE SUBROUTINE
        return
      END
```

```
***********************************************************************
***                    Start of subroutine SHPTHL                   ***
C  THIS IS ALMOST ENTIRELY BASED ON PAPE ALGORITHM FROM NETLIB.
***********************************************************************

      SUBROUTINE SHPTHL(J0)

      IMPLICIT DOUBLE PRECISION (A-H,O-Z)

      INTEGER LIM3
      PARAMETER (LIM3=15000)
      INTEGER NJ(LIM3), J0
      DOUBLE PRECISION INF,MJI,MJK
      DATA INF /9999999/

      COMMON /KSINPUT1/ NNODE, NUMORG,
     & NEXT(15000), NPRED(15000),
     & NTONODE(25000),NFMNODE(25000),XGUESS(25000),
     & GLINK(25000,2),SLINK(25000,4), CLINK(25000,6),NIPE(15000)

        COMMON /KSINPUT2/ ABI(15000),
     & D(2000,15000),NLINK, NPOINT(15000)

C     SHPTHL CALCULATES THE SHORTEST PATH LENGTHS (MJ) FROM A SPECIFIC
C     NODE (J0) TO ALL OTHER (N-1) NODES IN A NETWORK (FLIST,DFLIST,KF).
C     PREDECESSOR NODES ARE STORED IN WJ.
C
C     NTONODE      : FORWARD INDEX LIST
C     CLINK(K,1) : DISTANCE LIST
C     NEXT       : POINTER LIST FOR FLIST AND DFLIST
C     NNODE      : NUMBER OF NODES
C     CAB        : ARRAY OF SHORTEST PATH LENGTHS
C     J          : INITIAL NODE, FIRST NODE OF SHORTEST PATH
C     NPRED       : ARRAY OF PREDECESSORS FOR SHORTEST PATH CONSTRUCTION
C     NJ    : DOUBLE ENDED QUEUE FOR NODE DISCUSSION
C     INF   : A LARGE NUMBER
C
      DO 1 I=1,NNODE
        ABI(I)=INF
        NPRED(I)=0
          NJ(I)=0
  1   CONTINUE
      ABI(J0)=0.0D+0
C
C     I     : INDEX FOR NODE DISCUSSION, NODE UNDER DISCUSSION
C     NT    : POINTER TO THE END OF DEQUE NJ
C     MJI   : LOCAL VARIABLE OF MJ(I)
C     KFI   : LOCAL VARIABLE OF KF(I)
C     KFI1  : LOCAL VARIABLE OF KF(I)+1
C     IR    : INDEX FOR ARRAY DISCUSSION
C     K     : SUCCESSOR OF NODE I
C     MJK   : LOCAL VARIABLE OF MJ(K)
C     NJI   : LOCAL VARIABLE OF NJ(I), THE NEXT NODE OF NJ TO BE TAKEN
C             UNDER DISCUSSION
C
```

```
      NJ(J0)=INF
      I=J0
      NT=J0
C
C     OUTER LOOP
C     DISCUSSION OF NODES I
  2   KFI=NEXT(I)
      MJI=ABI(I)
        IF (I.EQ.1) THEN
        KFI1=1
        ELSE
        KFI1=NEXT(I-1)+1
      ENDIF
C
C *** INNER  LOOP
C *** DISCUSSION OF SUCCESSORS K
C
      IF (KFI1.GT.KFI) GO TO 6
      DO 5 IR=KFI1,KFI
        K=NTONODE(IR)
        MJK=MJI+CLINK(IR,1)
C *** NO DECREASE OF SHORTEST DISTANCES
        IF (MJK.GE.ABI(K)) GO TO 5
C *** DECREASE OF SHORTEST DISTANCES
          ABI(K)=MJK
C *** PREDECESSOR I OF NODE K
          NPRED(K)=I
C *** NODE K ALREADY IN THE DEQUE NJ ?
          IF (NJ(K))  4,3,5
C *** NODE K ADDED AT THE END OF THE DEQUE NJ
    3     NJ(NT)=K
          NT=K
          NJ(K)=INF
          GO TO 5
C *** NODE K ADDED AT THE BEGINNING OF THE DEQUE NJ
    4     NJ(K)=NJ(I)
          NJ(I)=K
          IF (NT.EQ.I) NT = K
    5 CONTINUE
C *** NODE I TAKEN FROM THE BEGINNING OF THE DEQUE NJ
    6 NJI=NJ(I)
      NJ(I)=-NJI
      I=NJI
      IF (I.LT.INF) GOTO 2
      RETURN
      END
```

```
*****************************************************************************
****                      Start of subroutine INDEXX                    ****
*****************************************************************************

      SUBROUTINE INDEXX

C This subroutine constructs an index table for the travel time between
C nodes in the network. From pp. 330-332 Numerical Recipes in FORTRAN.

      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
         INTEGER M,NSTACK,LIM3
         PARAMETER (M=7,NSTACK=50, LIM3=15000)

C Indexes an array ABI(1:NNODE), i.e., outputs the array NIPE(1:NNODE)
C such that ABI(NIPE(j)) is in ascending order for j = 1; 2; : : : ;
C  NNODE. The input quantities NNODE and ABI are not changed.

         INTEGER i,indxt,ir,itemp,j,jstack,k,l,
     &  istack(NSTACK)

      COMMON /KSINPUT1/ NNODE, NUMORG,
     & NEXT(15000), NPRED(15000),
     & NTONODE(25000),NFMNODE(25000),XGUESS(25000),
     & GLINK(25000,2),SLINK(25000,4), CLINK(25000,6),NIPE(15000)

         COMMON /KSINPUT2/ ABI(15000),
     & D(2000,15000),NLINK, NPOINT(15000)

         DO 11 j=1,NNODE
           NIPE(j)=j
 11   CONTINUE
      jstack=0
         l=1
         ir=NNODE
  1   IF(ir-l.lt.M)THEN
           DO 13 j=l+1,ir
             indxt=NIPE(j)
             a=ABI(indxt)
             DO 12 i=j-1,l,-1
               IF(ABI(NIPE(i)).le.a) GOTO 2
                 NIPE(i+1)=NIPE(i)
 12       CONTINUE
             i=l-1
  2       NIPE(i+1)=indxt
 13     CONTINUE
           IF(jstack.eq.0)RETURN
             ir=istack(jstack)
             l=istack(jstack-1)
             jstack=jstack-2
         ELSE
           k=(l+ir)/2
           itemp=NIPE(k)
           NIPE(k)=NIPE(l+1)
           NIPE(l+1)=itemp
           IF(ABI(NIPE(l)).GT.ABI(NIPE(ir)))THEN
```

```
            itemp=NIPE(l)
            NIPE(l)=NIPE(ir)
            NIPE(ir)=itemp
        ENDIF
           IF(ABI(NIPE(l+1)).GT.ABI(NIPE(ir)))THEN
          itemp=NIPE(l+1)
            NIPE(l+1)=NIPE(ir)
            NIPE(ir)=itemp
           ENDIF
           IF(ABI(NIPE(l)).gt.ABI(NIPE(l+1)))THEN
            itemp=NIPE(l)
            NIPE(l)=NIPE(l+1)
            NIPE(l+1)=itemp
           ENDIF
           i=l+1
           j=ir
           indxt=NIPE(l+1)
           a=ABI(indxt)
  3     CONTINUE
           i=i+1
           IF(ABI(NIPE(i)).lt.a)GOTO 3
  4     CONTINUE
           j=j-1
           IF(ABI(NIPE(j)).gt.a)GOTO 4
           IF(j.lt.i)GOTO 5
           itemp=NIPE(i)
           NIPE(i)=NIPE(j)
           NIPE(j)=itemp
           GOTO 3
  5     NIPE(l+1)=NIPE(j)
           NIPE(j)=indxt
           jstack=jstack+2
           IF(jstack.gt.NSTACK)PAUSE 'NSTACK too small in indexx'
           IF(ir-i+1.ge.j-l)THEN
            istack(jstack)=ir
            istack(jstack-1)=i
            ir=j-1
           ELSE
            istack(jstack)=j-1
            istack(jstack-1)=l
            l=i
           ENDIF
      ENDIF
         GOTO 1
c
         return
         END
```

```
**********************************************************************
****                      Start of subroutine STOCHO            ****
**********************************************************************

C *** THIS SUBROUTINE SOLVES A DIAL ASSIGNMENT.
C For more information, see pp. 122-129 and pp. 284-294 of Sheffi's
C "Urban Transportation Networks".

      SUBROUTINE STOCHO(JX,X)

      IMPLICIT DOUBLE PRECISION (A-H,O-Z)

      INTEGER LIMIT, LIM2, LIM3
      PARAMETER (LIMIT=25000, LIM2=2000, LIM3=15000)
      DATA INF /9999999/
      INTEGER BEGIN, JX
      LOGICAL NPOSSLST(LIMIT)

      DOUBLE PRECISION LK(LIMIT),
     &   W(LIMIT), SUM1(LIMIT),
     &   X(LIMIT), XSUM(LIMIT), ZERO, TEMP

      DOUBLE PRECISION P(LIM2,LIMIT), W1(LIM2,LIM2),
     &   SUMJD(LIM2), WJTD(LIM2,LIMIT), WJD(LIMIT),
     &   WOTI(LIMIT), WITJ(LIMIT)

      COMMON /KSINPUT1/ NNODE, NUMORG,
     & NEXT(15000), NPRED(15000),
     & NTONODE(25000),NFMNODE(25000),XGUESS(25000),
     & GLINK(25000,2),SLINK(25000,4), CLINK(25000,6),NIPE(15000)

      COMMON /KSINPUT2/ ABI(15000),
     & D(2000,15000),NLINK, NPOINT(15000)
      COMMON /KSINPUT2a/ THETA
      COMMON /KSINPUT3/ IDERIV,  NPOSS, NPOSSLST

      ZERO   = 0.0d+00

C *** PRELIMINARIES FOR SETTING UP SHEFFI VERSION OF
C *** SINGLE PASS OF DIAL ASSIGNMENT.

      K = 1
      BEGIN = 1
      DO 70 I = 1,NNODE
        DO 69 J = BEGIN,NEXT(I)
          IF (ABI(I).LE.ABI(NTONODE(J))) THEN
            LK(K) = EXP(theta*(ABI(NTONODE(J))-ABI(I)-CLINK(J,4)))
          ELSE
            LK(K) = ZERO
          ENDIF
            K = K + 1
   69   CONTINUE
        BEGIN = NEXT(I) + 1
   70 CONTINUE
```

```
C *******FIND LINK WEIGHTS - MODIFIED FORWARD PASS**************

      DO 77 I=1,NLINK
           W(I)=ZERO
           X(I)=ZERO
  77  CONTINUE
      DO 78 I=1,NNODE
           SUM1(I)=ZERO
           XSUM(I)=ZERO
  78  CONTINUE
         SUM1(JX)=1.0D+0
         DO 90 K=2,NNODE
           I=NIPE(K)

C ****** COMPUTE SUM(I) = SUM OF W(M,I)**
C ****** FIND LINKS GOING INTO I, ADD WTS.**

        DO 80 L=1,NLINK
          IF (NTONODE(L) .EQ. I) THEN
            W(L)=LK(L)*SUM1(NFMNODE(L))
                  SUM1(I) = SUM1(I) + W(L)
                 ENDIF
   80   CONTINUE
  90  CONTINUE

C ******* CALCULATE LINK VOLUMES - MODIFIED BACKWARD PASS************
C ******* CONSIDER NODES IN DESCENDING ORDER OF DIST. FROM ORIGIN, JX**

         TEMP=ZERO
      DO 110 K=0,(NNODE-2)
C ******* J = NODE NO.**
        J=NIPE(NNODE - K)
C ******* L = LINK NO.
        IF (SUM1(J).NE.ZERO) THEN
                TEMP=D(JX,J) + XSUM(J)
                IF (TEMP.NE.ZERO) THEN
            DO 105 L=1,NLINK
                 IF ( NTONODE(L) .EQ. J. AND.
     &                W(L).NE.ZERO ) THEN
                    X(L)= ( TEMP ) * ( W(L)/SUM1(J) )
                       XSUM(NFMNODE(L))=XSUM(NFMNODE(L))+X(L)
                 ENDIF
  105       CONTINUE
          ENDIF
        ENDIF
  110 CONTINUE
C
      RETURN
      END
```

```
**********************************************************************
****                      Start of subroutine MSA                 ****
**********************************************************************

      SUBROUTINE  MSA(YGUESS)

      IMPLICIT DOUBLE PRECISION (A-H,O-Z)

      INTEGER LIMIT,  LIM3
      PARAMETER (LIMIT=25000,LIM3=15000)

      LOGICAL NPOSSLST(LIMIT)

      INTEGER ITNUM, MAXMSA, phase, phaseit, minit

      CHARACTER*12 FNAME

      DOUBLE PRECISION ZERO, DIFF,
     & ERRMAX, ERRCRIT, FIT, YGUESS(LIMIT),
     & DA(LIMIT),XT(LIMIT),CONST(LIMIT)

      COMMON /KSINPUT1/ NNODE, NUMORG,
     & NEXT(15000), NPRED(15000),
     & NTONODE(25000),NFMNODE(25000),XGUESS(25000),
     & GLINK(25000,2),SLINK(25000,4), CLINK(25000,6),NIPE(15000)

      COMMON /KSINPUT2/ ABI(15000),
     & D(2000,15000),NLINK, NPOINT(15000)
      COMMON /KSINPUT2a/ THETA
      COMMON /KSINPUT3/ IDERIV,  NPOSS, NPOSSLST
      COMMON /KSINPUT4/ NUMB, NAME

      ZERO    = 0.0D+0

c     Instead of hard-coding termination criteria, read from a
c     file named 'termcrit'.  First line needs to be maximum number
c     of iterations before giving up, second line is maximum error
c     for a successful termination.  If file open fails, use hard
c     coded defaults.

      OPEN (UNIT=51,FILE='termcrit',STATUS='OLD',ERR=20)
      READ(51,*) MAXMSA
      READ(51,*) ERRCRIT
      read(51,*) minit
      GOTO 21
20    MAXMSA = 32000
      ERRCRIT = 1.0D+0
      minit = 20


C NOW DO MSA TO FIND INTIAL FEASIBLE SOL'N

  21  IF (NUMB.LE.2) THEN
        DO 22 K=1,NLINK
```

```
          IF (NPOSSLST(K)) THEN
            CONST(K)= ((( 1 / CLINK (K,2) )*
     &       ( (SLINK(K,1)/GLINK(K,2)) - 1 ) )**(2.5D-01))
          ENDIF
  22    CONTINUE
      ENDIF
      IF (NUMB.EQ.3) THEN
        DO 23 K=1,NLINK
          IF (NPOSSLST(K)) THEN
            CONST(K)= GLINK(K,2)
          ENDIF
  23    CONTINUE
      ENDIF
      ITNUM=0

      phase=1
      phaseit=0

  100 ITNUM=ITNUM+1
      IF (ITNUM.LE.MAXMSA) THEN
        DO 5 K=1,NLINK
          IF(NPOSSLST(K))THEN
              TEMP=(XGUESS(K)/CONST(K))-CLINK(K,3)
              YGUESS(K)=MAX(ZERO,TEMP)
          ENDIF
 5      CONTINUE

c FOR OTHER EXAMPLES (NOT TWIN CITIES)
c   INPUT FILES WERE FORMATTED DIFFERENTLY
c Other problems were used to test code originally.

        IF (NUMB.NE.3) THEN
          DO 110 K=1,NLINK
            CAP = CLINK(K,3)
            IF(NPOSSLST(K))THEN
              CAP = CAP + YGUESS(K)
            ENDIF
            VC = (XGUESS(K)/CAP)
            IF(NPOSSLST(K))THEN
              IF (YGUESS(K).GT.ZERO) THEN
                CLINK(K,4)=CLINK(K,1)*
     &                    (1.0D+0+1.5D-01*CONST(K)**4.0D+0)
              ELSE
                CLINK(K,4)=CLINK(K,1)*
     &                    (1.0D+0+1.5D-01*(VC)**4.0D+0)
              ENDIF
            ELSE
              CLINK(K,4)=CLINK(K,1)*
     &          (1.0D+0+1.5D-01*(VC)**4.0D+0)
            ENDIF
            DA(K)=ZERO
            XT(K)=ZERO
  110     CONTINUE
        ENDIF

c FOR TWIN CITIES PROBLEM
```

```
        IF (NUMB.EQ.3) THEN
          DO 140 K=1,NLINK
            CAP = CLINK(K,3)
            IF(NPOSSLST(K))THEN
              CAP = CAP + YGUESS(K)
            ENDIF
            VC = (XGUESS(K)/CAP)

C  CALCULATE TRAVEL TIMES FOR DIFFERENT FUNCTIONAL CLASSES

             IF(SLINK(K,2).EQ.9)THEN
c  FUNCTION TC[9] = T0
               CLINK(K,4)=CLINK(K,1)
             ELSE
               IF(SLINK(K,2).EQ.6)THEN
c  FUNCTION TC[6] = T0 * (2+SQRT(25*(1-(V/C))^2 + 1.266) - 5*(1-(V/C)) -
1.125)
                 CLINK(K,4)=CLINK(K,1)*(2.0D+0 +
     &                 ((2.5D+01*(1.0D+0-(VC))**2.0D+0 + 1.266D+0)
     &                 **5.0D-01)- 5.0D+0*(1.0D+0-(VC)) - 1.125D+0)
               ELSE
                 IF(SLINK(K,2).EQ.7)THEN
c  FUNCTION TC[7] = T0 * (2+SQRT(36*(1-(V/C))^2 + 1.210) - 6*(1-(V/C)) -
1.100)
                   CLINK(K,4)=CLINK(K,1)*(2.0D+0+
     &                   ((3.6D+01*(1.0D+0-(VC))**2.0D+0 + 1.210D+0)
     &                   **5.0D-01)- 6.0D+0*(1.0D+0-(VC)) - 1.100D+0)
                 ELSE
c  FUNCTION TC[1] = T0 * (2+SQRT(16*(1-(V/C))^2 + 1.361) - 4*(1-(V/C)) -
1.167)
                   CLINK(K,4)=CLINK(K,1)*(2.0D+0+
     &                   ((1.6D+01*(1.0D+0-(VC))**2.0D+0 + 1.361D+0)
     &                   **5.0D-01)- 4.0D+0*(1.0D+0-(VC)) - 1.167D+0)
                 ENDIF
               ENDIF
             ENDIF

          DA(K)=ZERO
          XT(K)=ZERO
  140     CONTINUE
        ENDIF



        DO 120 J=1,NUMORG
          CALL SHPTHL(J)
          CALL indexx
          CALL STOCHO(J,XT)
               DO 115 K=1,NLINK
  115              DA(K)=DA(K)+XT(K)
  120   CONTINUE
        ERRMAX=ZERO

        PHASEIT=PHASEIT+1
        IF (PHASEIT.GT.PHASE*MINIT) THEN
           PHASEIT=1
```

```
           PHASE=PHASE+1
        ENDIF

        FIT=FLOAT(PHASEIT)

        DO 130 K=1,NLINK
          DIFF=DA(K)-XGUESS(K)
          ERRMAX=DMAX1(ERRMAX,DABS(DIFF))
          XGUESS(K)=XGUESS(K) + DIFF/FIT
  130   CONTINUE

C *** ERRMAX IS CURRENT SOL'NS DIFF IN ASSIGNMENT FROM PREVIOUS,
C *** ERRCRIT IS DESIRED CONVERGENCE TOLERANCE.

             WRITE(3,*) 'ITNUM = ', ITNUM, ', ERRMAX = ', ERRMAX
c            PRINT *, 'ITNUM = ', ITNUM, ', ERRMAX = ', ERRMAX
             FNAME= 'rslt'//CHAR(48 +INT(ITNUM/1000))//
     &             CHAR(48 +INT(MOD(ITNUM,1000)/100))//
     &             CHAR(48 +INT(MOD(ITNUM,100)/10))//
     &             CHAR(48 +MOD(ITNUM,10))//'.txt'
             OPEN (UNIT=4,FILE=FNAME,STATUS='unknown')
             WRITE(4,*) 'ITNUM = ', ITNUM, ', ERRMAX = ', ERRMAX
         do 12 k=1,NLINK
           write(4,'(2e20.11)')xguess(k),YGUESS(k)
   12    CONTINUE
         close (4)
        IF (ERRMAX.GT.ERRCRIT) GOTO 100
           ENDIF
             WRITE(3,*) 'FINAL = '
         do 11 k=1,NLINK
           CAP=CLINK(K,3)
           id=0
           IF (SLINK(I,1).EQ.1.OR.SLINK(I,1).EQ.2) THEN
             id=1
           endif
           IF(NPOSSLST(K))THEN
              CAP=CAP+YGUESS(K)
           ENDIF
           VC = (XGUESS(K)/CAP)
c           write(3,'(I8.2,2F12.0,1F12.5,I8.2)') k, xguess(k),
c     &       YGUESS(k), VC, id
           write(3,'(2e14.5)')xguess(k),YGUESS(k)
   11    CONTINUE

C *** DONE WITH MSA SUBROUTINE
      END
```

Appendix C: Sample Input Excerpts

Origin-Destination Demand

256342 ← [Total O-D Pairs]
1,1,1422.52 ← [Origin], [Destination], [Demand]
1,2,142.54
1,3,229.24
1,4,2.94
1,5,16.61
1,6,13.20
1,7,21.33
1,8,4.80
1,9,41.70
1,10,9.60
1,11,29.63
1,12,16.52
1,13,7.88
1,14,4.39
1,15,3.63
1,21,3.37
1,23,2.26
1,24,5.40
1,25,3.48
1,26,19.55
1,27,14.03
1,28,6.61
1,29,3.77
1,30,19.32
1,31,5.88
1,32,4.07
1,33,5.82
1,34,12.72
1,35,5.25
1,36,17.21
1,37,5.67
1,39,28.54
1,40,43.69
1,41,34.90
1,42,12.45
1,43,69.02
1,44,13.45
1,45,25.80
1,46,21.88
1,47,7.85
1,48,8.57
1,49,12.96
1,50,14.34
1,51,2.72
1,52,2.31
1,53,6.45
1,55,2.52
1,56,2.36
1,57,2.66
1,58,3.62
1,59,2.33
1,60,2.37
1,63,2.77

Network Links

```
3
'TWIN CITIES PM 2030 LOS C'
1236,12598,21158
2,1,2
0.68
1,7738,9,2.3000,23.0000,1,1,0
1,9023,9,1.4400,22.9800,1,1,0
2,7736,9,1.9500,22.9900,1,1,0
2,8044,9,2.1600,23.0200,1,1,0
2,8045,9,0.8600,23.0400,1,1,0
3,7741,9,2.4200,23.0100,1,1,0
3,9016,9,1.6300,23.0100,1,1,0
3,9017,9,1.7300,23.0200,1,1,0
3,9023,9,1.7400,23.0000,1,1,0
4,7764,9,1.1100,22.9700,1,1,0
4,9016,9,2.0700,23.0000,1,1,0
4,9018,9,1.5900,22.9900,1,1,0
5,7711,9,0.4800,23.0400,1,1,0
5,7741,9,1.3400,22.9700,1,1,0
6,7743,9,0.9500,22.9800,1,1,0
6,9017,9,1.5300,23.0100,1,1,0
6,9018,9,1.4900,22.9800,1,1,0
7,7712,9,1.3200,23.0200,1,1,0
7,7735,9,0.4600,23.0000,1,1,0
8,7660,9,0.1000,23.0800,1,1,0
8,7731,9,0.8900,23.0200,1,1,0
9,7734,9,2.3300,22.9900,1,1,0
9,7735,9,2.0400,23.0100,1,1,0
9,8045,9,2.1800,22.9900,1,1,0
10,7728,9,1.3600,22.9900,1,1,0
10,7733,9,0.6200,22.9600,1,1,0
11,7725,9,0.9000,22.9800,1,1,0
11,8046,9,0.7600,23.0300,1,1,0
12,7725,9,2.0000,22.9900,1,1,0
12,7729,9,1.2500,23.0100,1,1,0
12,9244,9,2.6200,23.0200,1,1,0
13,7724,9,2.5600,22.9900,1,1,0
13,7728,9,1.9900,23.0100,1,1,0
13,7730,9,0.9100,23.0400,1,1,0
14,7544,9,1.4700,23.0300,1,1,0
14,7724,9,0.6100,23.0200,1,1,0
15,7541,9,1.3700,23.0300,1,1,0
15,9244,9,2.6900,22.9900,1,1,0
16,7540,9,1.1800,22.9900,1,1,0
17,7535,9,1.1900,23.0300,1,1,0
17,7540,9,1.3000,23.0100,1,1,0
18,7535,9,1.9000,22.9800,1,1,0
19,7534,9,0.8500,22.9700,1,1,0
20,7667,9,0.3500,23.0800,2,1,0
20,7670,9,0.5000,23.0800,2,1,0
21,7536,9,2.5200,23.0100,1,1,0
21,7669,9,1.9200,22.9900,1,1,0
21,7673,9,0.4800,23.0400,1,1,0
21,7678,9,1.2400,23.0300,1,1,0
```

Annotations (arrows pointing to the lines above):

- Problem ID Number and Name (lines 1–2)
- Number of Origins, Nodes, and Links (line 3)
- Number of Road Types to expand and their functional classes IDs (line 4)
- Level of Service Desired (line 5)
- [Origin], [Destination], [Functional Class], [Link Length], [FFS in mph], [Area Type], [Number of Lanes], [Capacity] (link records)

Transportation Analysis District (TAD) Aggregation

```
1236
1,1,1
2,1,1
3,2,3
4,2,3
5,3,5
6,2,3
7,3,5
8,3,5
9,3,5
10,4,10
11,4,10
12,4,10
13,4,10
14,4,10
15,5,15
16,9,16
17,9,16
18,9,16
19,8,19
20,8,19
21,8,19
22,8,19
23,8,19
24,8,19
25,8,19
26,7,26
27,7,26
28,7,26
29,7,26
30,7,26
31,11,31
32,11,31
33,11,31
34,10,34
35,7,26
36,7,26
37,7,26
38,10,34
39,6,39
40,6,39
41,6,39
42,6,39
43,6,39
44,6,39
45,6,39
46,6,39
47,6,39
48,6,39
49,10,34
50,10,34
51,10,34
```

Number of Transportation Analysis Zones (TAZs)
[TAZ], [TAD], [TAZ representing the given TAD]