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BAYESIAN METHODS FOR ESTIMATING AVERAGE VEHICLE CLASSIFICATION VOLUMES

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Technical Report Documentation Page

1. Report No.	2.	3. Recipient□s Accession No.
MN/RC - P2000-02		

4. Title and Subtitle	5. Report Date
BAYESIAN METHODS FOR ESTIMATING AVERAGE VEHICLE CLASSIFICATION VOLUMES	October 1999
	6.
7. Author(s)	8. Performing Organization Report No.
Gary A. Davis, Ph.D. Shimin Yang	
9. Performing Organization Name and Address	10. Project/Task/Work Unit No.
University of Minnesota Department of Civil Engineering 500 Pillsbury Drive, S.E. Minneapolis, MN 55455	11. Contract (C) or Grant (G) No. c) 74708 wo) 26
12. Sponsoring Organization Name and Address	13. Type of Report and Period Covered
Minnesota Department of Transportation 395 John Ireland Boulevard St. Paul Minnesota, 55155	Staff Paper – 1992 to 1998
	14. Sponsoring Agency Code

15. Supplementary Notes

As a staff paper, this publication is intended for internal use by the Minnesota Department of Transportation (Mn/DOT). Distribution is limited.

16. Abstract (Limit: 200 words)

This report describes the development of a data-driven methodology for estimating the mean daily traffic (MDT) for different vehicle classes from short classification-count samples. Implementation of the methodology requires that an agency maintain a small number of permanent classification counters (PCC), whose output is used to estimate parameters describing their monthly and day-of-week variation patterns and covariance characteristics. The probability of a match between a short classification count sample and each of the PCCs is computed, as well as the estimates of the short-counts site's MDTs which would arise if the short-count site had variation patterns identical to each of the PCCs. The final MDT estimates are then simply the weighted averages of these component MDTs, with the matching probabilities providing the weights.

Empirical evaluation of the methods using data collected at the Long Term Pavement Performance Project sites in Minnesota indicated that a reliable match of a short-count site could be made using a sample consisting of a one-day classification count from each month of the year. An evaluation of two-day classification count samples indicated that a two-day count is not sufficient to reliably match the site to a factor group, justifying estimation of MDT using weighted averages. For estimating combination vehicle MDT, these samples should be taken between May and October, and between Tuesday and Thursday. In this case the estimated MDT differed on average by about 10% - 12% compared to estimates based on full year's worth of counts, and differed by less than 26%, 95% of the time.

17. Document Analysis/Descriptors		18. Availability Statement
MDT (mean daily traffic) PCC (permanent classification counter)	traffic data vehicle classification bayesian method	

19. Security Class (this report)	20. Security Class (this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	137	

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Final Report

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October 1999

Published by:

Minnesota Department of Transportation
Office of Research & Strategic Services
Mail Stop 330
395 John Ireland Boulevard
St. Paul, MN 55155

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EXECUTIVE SUMMARY

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CHAPTER 1
INTRODUCTION

1.1 Project Motivation

Of all the data collection programs carried out by state agencies, traffic data are the most used and most important in virtually every decision-making process [1]. A large amount of traffic data is needed for the purposes of transportation analysis, roadway design, pavement maintenance and operational activities. One of the most essential data items is traffic volume, broken down by vehicle class. The use of traffic counts and vehicle classification data to develop estimates of average annual daily traffic (AADT) and vehicle miles of traveled (VMT) measures is a common practice throughout the United States [2]. In the *AASHTO Guidelines for Traffic Data Programs*, forty-two different uses are identified for the estimates of AADT and fifteen uses for estimates of vehicle classification percentage. Traffic classification volume estimates, mainly truck volume estimates, play vital roles in roadway and pavement projects because pavement designs are based on the cumulative equivalent standard axle load (ESAL), and traffic system operating conditions are accessed using Level of Service (LOS) [3].

In the past, the limitations in personnel and equipment have prevented vehicle classification data from being collected in all seasons of the year, so much existing literature on highway traffic patterns tends to focus on total vehicle traffic rather than classification traffic volume [3]. Consequently, until recently, there was little understanding of how truck travel changes seasonally, from month to month, or from day to day, on the highway system [4]. Nowadays, equipment such as automatic vehicle-classifier (AVC) and weigh-in-motion (WIM) devices enable us to obtain continuous records of traffic volume with respect of vehicle type. However, due to economical constraints and equipment shortage, it is still not feasible to obtain continuous data collection on all roadways in a given jurisdiction. In practice, highway agencies use portable counters to collect seasonal and short-period data, ranging from several hours to several weeks, from the locations of interest. Davis [5], in a review of published studies on estimating AADT, concluded that reasonably accurate estimates can be achieved by adjusting for seasonal and day of week variation, as long as the correct adjustment terms are known beforehand. However, incorrect adjustment can lead to substantial errors. There has been less work on the case of truck volume and classification statistics from short-term traffic counts [3]. In Stamatiadis and Allen [2], a survey was conducted of current uses of vehicle classification data and methods used for data collection throughout the United States. The results indicated that most states use no seasonal adjustment factors nor are they planning to develop any factors in the near future. At the same time, a number of states expressed a concern about the difficulty of developing seasonal and adjustment factors as well as the reliability of these factors over time. From there, we can see that although it is a well recognized fact about the importance of vehicle classification volume estimates in the modern highway activities, the problem on how to obtain the best estimates of AADT by vehicle classification from short period counts has not received fair investigation. Thus, there is an imperative need to study classification volumes in order to have a better understanding of the seasonal and day-of-week patterns of classification volumes and obtain reliable estimates of classification average annual daily traffic volume from short-period counts.

1.2 Recent Work On Estimation of AADT by Vehicle Classification

As noted earlier, the main objective of most highway traffic counting programs is to obtain estimates of those quantities used in transportation planning and design activities. Since it is not feasible to place permanent, year-round traffic counters on every highway roadway segment, those important estimates are often calculated from short-period counts. Short count collection period can range from several hours to 2 weeks.

A continuous count of traffic at a section of a road will show that traffic volume varies from day to day and from month to month. However, regular observation of traffic volumes over the year shows that although they vary, this variation is repetitive and rhythmic [6,7]. Therefore, it will be of benefit if we understand the relationship between traffic volumes in one season or month of the year and those for the entire year. In most states of the United States and provinces in Canada, a small number of year-round permanent traffic recorders are deployed in a statewide range, and these are used to obtain bias-corrected estimates of AADT from short samples via the so-called factor group idea. The following steps are involved in this bias-corrected estimation of AADT. First, data are collected from the permanent traffic recorders, and grouped together according to their similar monthly and day-of-week variation patterns. Second, the monthly and day-of-week adjustment factors for each group are computed. Third, short-period counts are made at the roadway segment of interest and it is assigned to one of the factor groups. Finally, the monthly and day-of-week adjustment factors characterizing the group the short count site belongs to, are used to produce an estimate of the annual average traffic volume. Several previous studies have looked into expanding this method to AADT by vehicle class.

Satish Sharma and his associates at the University of Regina have applied the above idea to conduct the most extensive data-driven investigation on the estimates of AADT from short-period counts. In Sharma and Allipuram [8], a hierarchical grouping technique was applied to data from 61 ATR sites in the province of Alberta to generate seven seasonal factor groups, each group being characterized by 12 ratios of monthly ADT to AADT. In Sharma and Leng [9], ATR data from 52 sites in the state of Minnesota were clustered into four seasonal factor groups, based on monthly variation patterns. In both studies, estimates of the monthly correction factors were computed for the months during which the short counts were made and then compared to the monthly factors characterizing the factor groups. The site was then assigned to that group having the closest monthly factors. Later on, Sharma and his associates extended their work to the estimation of truck annual average daily traffic (TAADT) using 48-hour short counts from portable automatic vehicle classifiers (AVC) [3]. The monthly and daily truck volume adjustment factors were calculated in the same manner as the volume adjustment factors are calculated for the estimation of AADT in the previous work. Their research was conducted using three truck classes: single-unit, single-trailer and multi-trailer. They also investigated the effect of estimation errors in several scenarios on the constitution of factor groups using simulation methods. They concluded that reasonable estimates of TAADT can be obtained only for one scenario, when factors used to expand the short-period data come from a permanent AVC site that has a truck traffic pattern that is similar to the one at the short count site.

Another investigation of the truck volume pattern and the estimation of average annual daily truck traffic using short duration counts was done by Mark Hallenbeck and his assistant Soon-Gwam Kim in 1993 [10]. Their study was based on the truck volume data collected by the Washington State Department of Transportation (WSDOT) over four and one half years, from 1988 to 1993. The majority of the data were collected using 4-bin vehicle length classifiers. The project team assumed that Length Bin 1 equals Bin 1,2 and 3 in FHWA 13 Axle Bin classifications, Length Bin 2 equals Axle Bins 4,5,6 and 7, Length Bin 3 equals Axle Bins 8,9 and 10, and Length Bin 4 equals Axle Bins 11,12 and 13. Their work was based on previous studies examining the pattern in truck volumes and weights across the nation. For example, a study by the state of Minnesota showed that the use of "traditional" automobile factoring procedures for calculating and applying seasonal factors to short-duration truck counts was inappropriate and often led to increased error in the estimation of average annual truck volumes. Thus, four further analyses were performed in their research with the length classifier data. First, an effort was made to establish the truck volume patterns using the data collected from the permanent sites. The project team computed volumes by truck class for each site for each average annual day, average weekday and weekend day for each month, and for the average day of each month. The methodology recommended in the *AASHTO Guidelines for Traffic Data Programs*, rather than the traditional simple averaging method was used to account for the bias caused by differing number of weekdays/weekends present in each month. For example, the Monthly Average Weekday Traffic (MAWDT) for each category calculates the average weekdays (Tuesday through Thursday) for each month and then computes the average weekday for the month as the simple average of these three average days. Hallenbeck and Kim obtained the Annual Average Daily Traffic (AADT) by vehicle class by first computing the average day of the week for the year (e.g., the average Monday for the year is the arithmetic mean of the 12 average Mondays for each month), then the arithmetic mean of those seven values was calculated to get AADT estimates. The 12 ratios of MAWDT over AADT were calculated and plotted for each vehicle class, and for each site to investigate the truck seasonal volume pattern. Those graphs revealed that first, the truck volumes varied sufficiently over the year and truck volume patterns were significantly different from the automobile patterns and second, the four vehicle classes had very different seasonal patterns, regardless of the volume or functional classification of the roadway or the geographic location of the site. In general, the longer truck categories show less seasonal variation than the short truck and automobile classifications. The second objective in their project was to develop factor groups and associate a short count site to one of the factor groups. They tried several alternatives though none of them performed as well as desired. They first employed a pictorial approach by visually matching the volume patterns for different sites. Then they tried a cluster method, but it was hard to produce a usable methodology with this approach. Finally they used a multiple regression approach to define factor groups, but the results were still not impressive. The third objective of their research was to address the question of whether seasonal adjustments specific to each truck class were necessary to accurately estimate annual average daily truck volumes. Their findings showed that, for every truck category, estimation errors were considerably reduced for the short count sample when the seasonal adjustment was applied. Then they explored the impact on accuracy of different count durations on the annual volume estimates. They investigated first the estimation error of factoring as to the different count durations, including individual weekdays and the combination weekdays. The monthly factors (MAWDT/AADT) for each site were computed and used to convert the short counts from that same site to estimates of AADT, so that there was no factor group assignment error. It showed that for Length Bin 1, the average error in the estimation of annual volume ranged from 6% to 9%, and 95% of all the estimates were within the 18% of the actual annual volume. Bin 3 and Bin 4 had the highest estimation error and mean errors ranged from 9% to 23%, and 95% of all the estimates were within 60 % of the actual annual volume. It is also shown that a 3-day count generally provided a more accurate estimate of annual average volumes than a single day count, but was only marginally better than the 2-day estimates. The last test they did was to increase the count duration as well as to collect data at multiple times in the course of the year and then average the counts to obtain the annual estimates. It was shown in their work that the estimates based on four week-long counts were able to perform 1.4 to 2 times better than the factoring method mentioned above. Hallenbeck and Kim also had some weigh-in-motion

(WIM) scale data available for examining the aggregated four-bin classification scheme versus the FHWA 13-bin axle-based classifications. They found that, when disparate vehicle class patterns are combined into fewer categories, the individual peaks are "dampened". However, the "dampening" effect makes the seasonal factors for larger vehicle categories more stable and thus more capable of predicting total traffic volume.

Another systematic study of the estimates of AADT by vehicle class was reported in the *Use of Data from Continuous Monitoring Sites* [11]. In the report, authors first reviewed current procedures for collecting traffic data and for using these data to estimate AADT, VMT, AADT and VMT by vehicle class. They also performed an extensive analysis of data from continuous automatic traffic recorders in four states, producing recommendations relating to the collection and analysis of traffic data. Among other things, they suggest that short-duration classification counts be collected over periods that are multiples of 24 hours and be at least 48 hours long. They also compared the effect of applying two alternative factoring procedures to simulated 48-hour counts, compared to using no adjustment. The first factoring procedure used a version of "current-year" factoring, in which factors are derived entirely from actual ATR data containing the date of the short count; while the other procedure uses a "historic" factoring, in which factors are derived from previous ATR data. The results show that current-year factoring produces substantially better AADT estimates than are obtained when unfactored 48-hour counts are used. It also produced appreciably better estimates of AADT than historic factors. Thus they conclude that, first, good factoring procedures could produce AADT estimates that are substantially better than those produced by using unfactored counts; second, it is recommended to use historic factors developed from actual and imputed data for the preceding calendar year if the use of current-year factors are not available, which is often the case.

They proposed seven approaches for estimating AADT by vehicle class (AADTVC). The first is to collect classification counts for the entire year and then use the AASHTO method to get AADTVC, which is obviously the most accurate way, but only feasible for a relatively limited number of sites. The second and third approaches can only be used at sites that are near the permanent AVC site, because they require seasonal and weekly factoring using the permanent site. The authors suggest either collecting short-duration classification counts and VC directly, or collecting the total traffic volume and factoring them to get AADT, and then distributing AADT across vehicle classes. A fourth approach uses unfactored short-duration classification counts. Such unfactored counts do not provide very good estimates of AADTVC because truck volumes have seasonal and weekly fluctuations that differ from those of passenger cars. For example, unfactored weekday truck counts normally tend to overestimate AADT by trucks. Approaches five and six are similar to approaches two and three, but can be applied to sites that are not near a permanent AVC on the same road, or on another roads, but little is mentioned about how to assign the site to a factor group. Approach seven suggests collecting several short-duration classification counts over the course of a year and averaging these counts to produce AADTVC. Results show that approaches four to six cannot produce estimates of AADTVC as good as those produced by the first three approaches and the last one is much more expensive and thus less attractive.

This review raises several unresolved issues. First, Hallenbeck and Kim investigated thoroughly the seasonal truck volume pattern using the MAWDT/AADT ratio, but little was done concerning weekly trend. Also they did not investigate the relation between sample size and estimation error in detail. Second, both Hallenbeck and Kim, and Cambridge Systemetics recommended the seasonal adjustment of short duration count using factor group assignment, but the way they made the factor group assignments was subjective and would have many changes if used in other states. Thus little progress has been made toward a usable methodology for assignment. Third, all the tests and thus the results presented are based on the "best scenario"which masks the very critical issue of the factor group assignment. For example, in Hallenbeck and Kim, a monthly factor was computed and then used to convert short counts from that same site. In the Cambridge Systemetics research, it was suggested to use the adjusting factors from the permanent AVC site near the short count site, which assumes that they have similar seasonal and weekly patterns. This is also true of Sharma's data-driven work in the estimation of TAADT. Thus they do not provide a usable treatment for the cases where the assignment to a factor group is ambiguous, even though they all agree and emphasized that if the adjustment factors came from a permanent traffic counter which is not near the short count site or reflects only a general traffic pattern rather than similar truck traffic variations, the estimation errors could be very large. Finally, the ultimate goal of the traffic sampling is to generate predictions of mean daily traffic or total traffic for the year by vehicle class based on the samples. The methods mentioned above provide little guidance on how to generate these predictions using a statistical model describing the traffic counts.

An investigation into using short-period counts to obtain AADT was made by Gary Davis and Yuzhe Guan at the University of Minnesota [12]. 48 ATRs which were formed into three factor groups were used in this research. They solved the assignment problem by applying Bayesian decision methods to a lognormal model of traffic counts to compute the posterior probability that a non-ATR site belonged to a factor group, given a count sample. The Bayesian approach was also used to develop a heuristic method for identifying which days to count so as to minimize the likelihood of assigning a site to the wrong factor group, as well as an empirical Bayes (EB) estimator of MDT. But it only solved estimation of AADT in total, which means in the univariate case. Since the estimation of truck volumes using short period count is also important, extending this work to handle the traffic volumes by vehicle type is of great significance.

1.3 Objectives

The chief objective of this study is to develop a sound, data-driven methodology for estimating the mean daily traffic of vehicle classes from short classification count samples. This will require us to (1) investigate the seasonal and weekly pattern of truck volumes, (2) develop a statistical model describing classification counts, (3) develop a method for classifying short-period count sites as to factor group for classification counts, (4) perform Bayesian estimation of MDT for multiple vehicle categories and (5) investigate the impact of the duration of short count and sampling plan on the precision of estimates of mean daily traffic by vehicle class.

CHAPTER 2

A STATISTICAL MODEL OF DAILY CLASSIFICATION TRAFFIC COUNT

Daily classification traffic counts provide data for estimating MDT by vehicle class, therefore, the first important task is to develop an appropriate class of probability models and a corresponding likelihood function, for the classification counts. Afterwards, all inference about MDT will be based on the statistical model. In the *Traffic Monitoring Guide*, the AASHTO *Guidelines for Traffic Data Programs* and in some other research reports, the clustering and sample size computation procedures assume that traffic counts are normally distributed, although the connection between derived quantities, such as computed monthly factors or percent errors, and the underlying traffic counts is left unspecified. Other than these, there is no source which appears to have determined an appropriate class of likelihood functions for daily classification traffic counts. Thus, before starting our Bayesian estimation work, it is necessary to develop a plausible statistical model and corresponding likelihood function for daily classification traffic counts.

2.1 Description of Dataset

To support the model development and the research that will be carried out thereafter, raw monitored data files recorded by permanent WIM in the state of Minnesota were obtained from the Central Traffic Database (CTDB) of the Long Term Pavement Performance Project (LTPPP) [13]. Using an Excel Macro program, 60 daily classification count data sets were extracted from the raw data files, covering 15 WIM sites for the years 1992 to 1995. These classification counts are by-lane, by-direction counts and use Scheme F, a 15-class categorization defined in the *Traffic Monitoring Guide* [14]. Those daily counts that are absent or incalculable are coded as missing and no imputation is done for them. It turned out that large blocks of missing data were found for some of WIMs, and deleting them from the data set left a total of 24 set of yearly WIM classification counts coming from 10 separate sites. Specifically, we have 8 WIMs in the year of 1992, 6 in

1993, 7 in 1994 and 3 in 1995. Finally, a code specifying holidays and long weekends was assigned to each daily classification count to indicate whether or not that count was subjected to holiday effects.

We have also obtained some supplementary information about the sites such as the SHRP four digit site identification code, the route number where the site is located and the Milepost of the site as well as the location of the site. Table 2.1 summarizes information about the sites and Figure 2.1 displays a map showing their approximate locations.

2.2 Model Development

As stated earlier, daily traffic volumes tend to show seasonal and day-of-week variation patterns. A convenient and traditional starting point for specifying a classification daily count model is to assume that the effect of seasonal and day-of-week variations on the daily classification count is multiplicative, and has the form,

$$E\left[z_t^{(k)}\right] = z_0^{(k)} M_i^{(k)} W_j^{(k)} \qquad (2.1)$$

where,

$z_t^{(k)}$ = traffic volume on day t for count class k, during month i and day of week j,

$z_0^{(k)}$ = mean daily traffic for count class k,

$M_i^{(k)}$ = monthly multiplier accounting for the deviation of month i from MDT for count class k,

$W_j^{(k)}$ = day-of-week multiplier accounting for the deviation of day-of-week j from MDT for count class k.

Daily traffic counts are not continuous variables. However, researchers and practicing statisticians have adapted the powerful linear regression methodology to different data configurations because of the simplicity and widespread use of linear models. Generalizing the linear model to discrete

Table 2.1 WIM Site Information

SITE ID	ROUTE	MILEPOST	DIRECTION/LANE of GPS*	# OF LANES in Rdw	LOCATION	YEAR	DATA AVAILABLE
1019	US169	179.5	NORTH-1	4	PRINCETON	1992	331
						1994	352
						1995	362
1023	US 2	115.55	EAST-1	4	BEMIDJI	1992	359
						1993	341
						1994	351
1029	ST 65	36.17	NORTH-1	4	.27 Mi. N. of CSAH 5 in ISANTI	1992	355
						1994	363
1085	ST 16	203.34	EAST-1	2	3.8 Mi. E. of I 90	1992	366
						1995	354
4033	I 35 E	89.76	NORTH-1	4	BURNSVILLE	1992	359
						1993	289
						1994	334
4037	I 35 E	89.76	SOUTH-1	4	BURNSVILLE	1993	288
						1994	337
4040	US 2	172.21	WEST-1	4	DEER RIVER	1992	364
						1993	336
4055	I 94	174.51	EAST-1	4	CLEARWATER	1994	343
6251	US 2	115.55	WEST-1	4	BEMIDJI	1992	359
						1993	345
						1994	351
9075	US 71	103.45	NORTH-1	2	.4 Mi. N. of CSAH 11 of OLIVI	1992	347
						1993	343
						1995	334

* GPS lane means General Pavement Study lane and lane 1 is the outter lane in each direction.

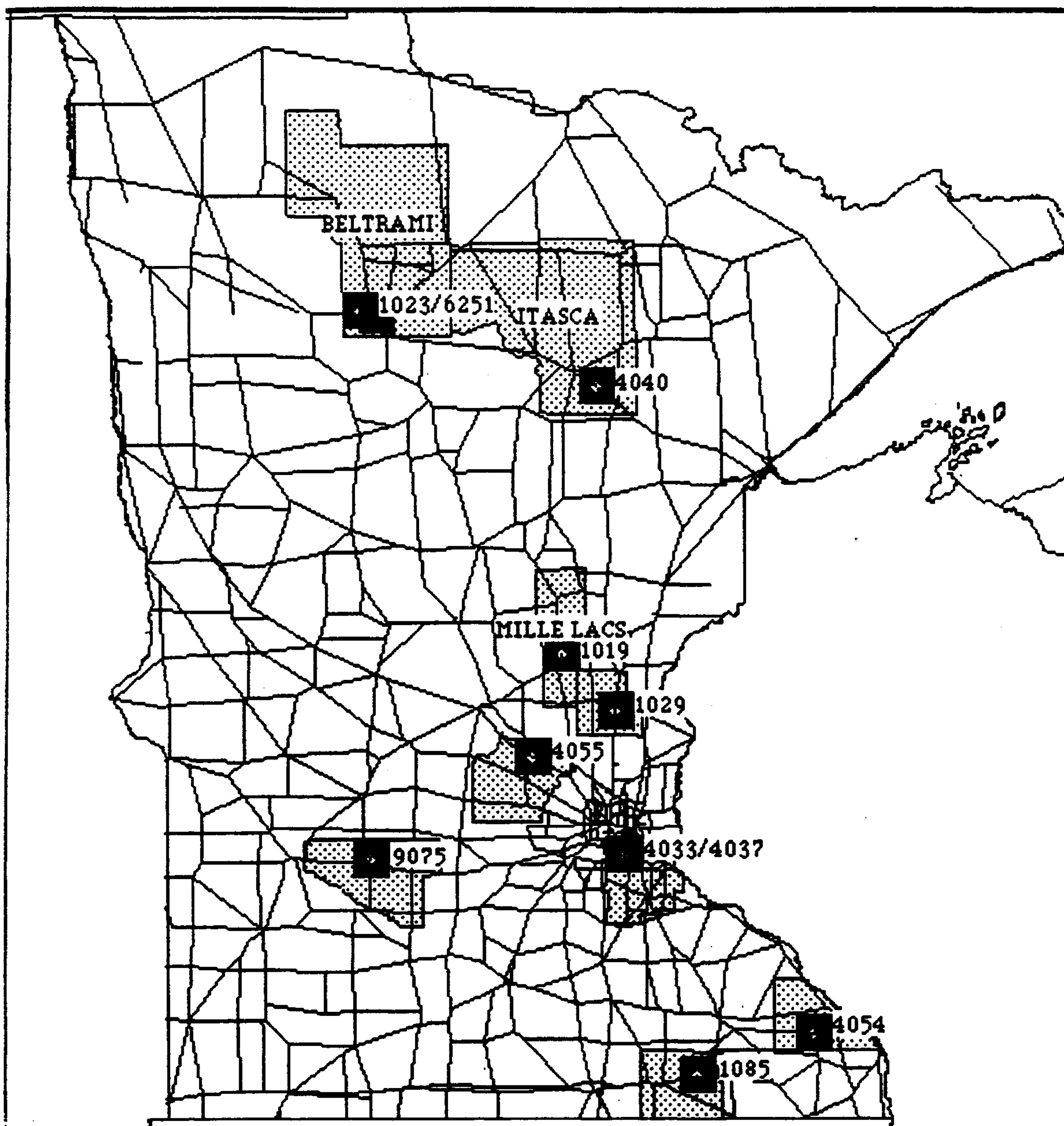


Figure 2.1 LTPP Site Location

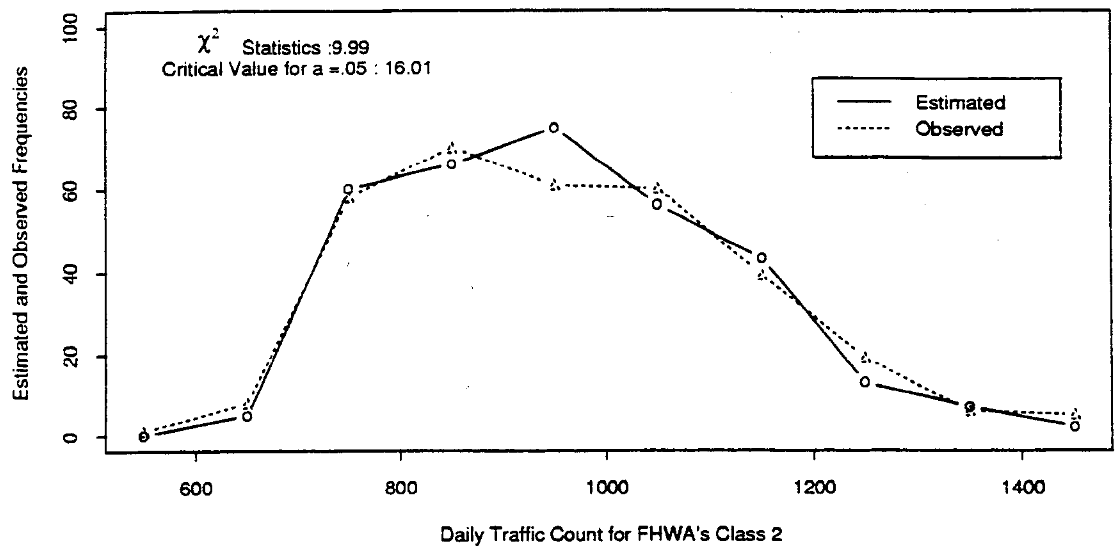
response variables can be done using generalized linear models, developed by Nelder and Wedderburn in 1972 [15]. Also the daily classification traffic count model in (2.1) can be transformed into a loglinear model by taking the logarithms of both sides of the equation, and this produces a linear relationship between the logarithm of the expected classification daily traffic count and the monthly and day-of-week variation effects. The probability distributions which support convenient analysis of loglinear models such as the Poisson, Gamma, Negative Binomial and the Lognormal model can be applied to the count variable. Here are some results from the preliminary analysis of those statistical models.

2.2.1 Poisson Model

Count data are frequently modeled using a Poisson distribution model, so our first trial is to fit a Poisson regression model using generalized linear modeling. This was done using the S-Plus *glm* function. Count data of Passenger Car (FHWA class 2) and Two-Axle, Six-Tire, Single-Unit Trucks (FHWA class 5) were selected to test the Poisson model. We then calculated the residuals, and "standardized" residuals using the Poisson model's standard deviation, as well as the expected and observed frequencies of number of days for which daily counts fall into certain ranges. Graphs depicting the estimated and observed frequencies, the standardized residuals and the estimated autocorrelation functions for the Poisson model, using counts of site 1023 in 1992, are shown in Figure 2.2--Figure 2.4. We found that the autocorrelation plots for the residuals tended to show serial correlation and that, the "standardized" residuals were not distributed with a variance of 1.0, but rather showed greater day-to-day variability. The larger truck classes showed greater day-to-day variability. This suggests that an overdispersed model would be a better choice than a Poisson model.

2.2.2 Negative Binomial Model

We then tried to fit a Negative Binomial model, which is a straightforward model allowing for overdispersion through generalized linear modeling methods. The maximum likelihood estimates



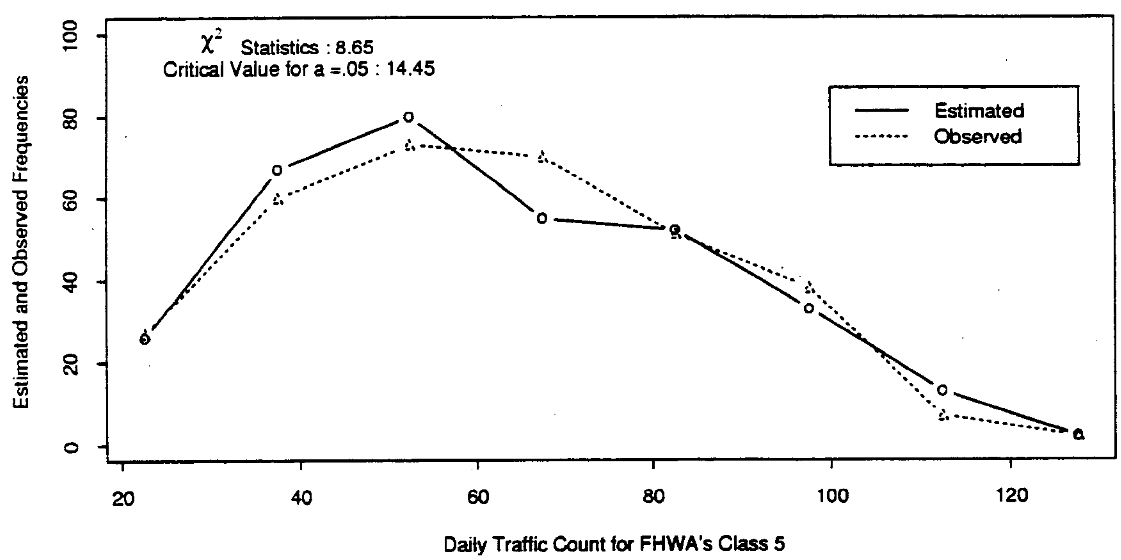
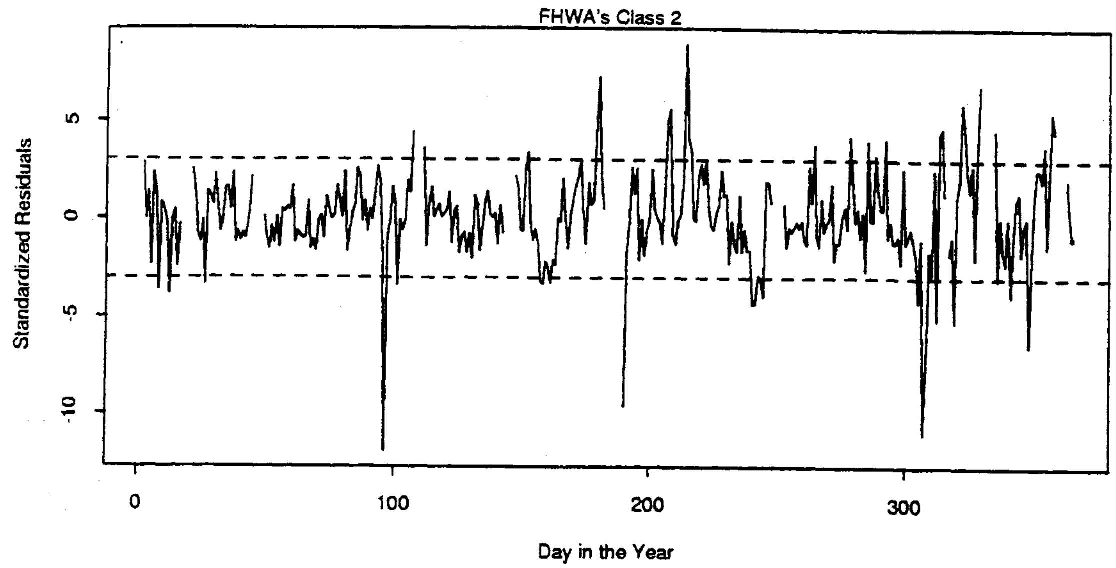


Figure 2.2 Estimated and Observed Frequencies for Poisson Model using Site 1023 (92)



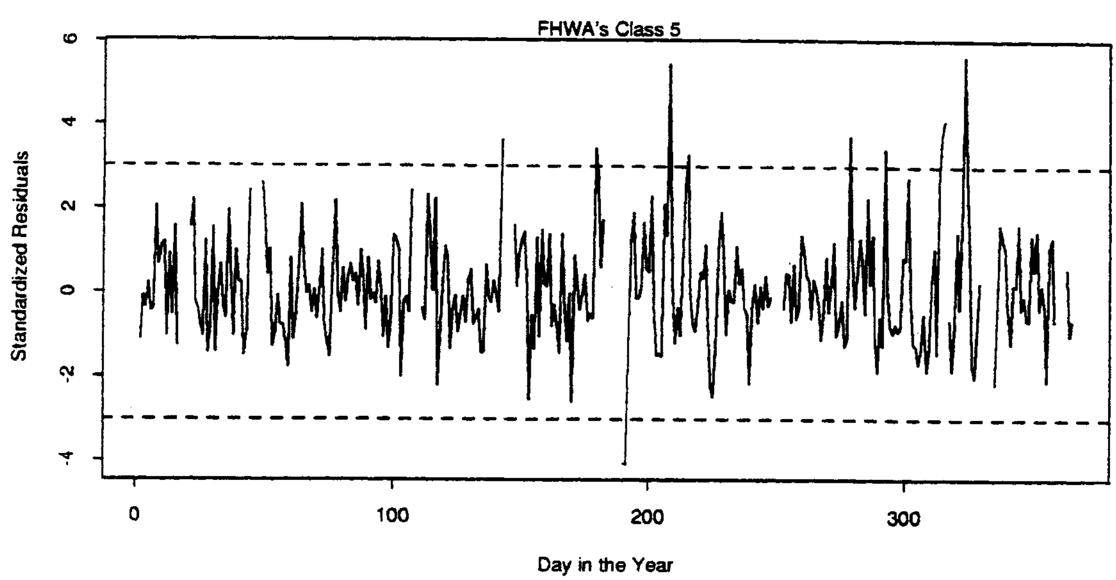
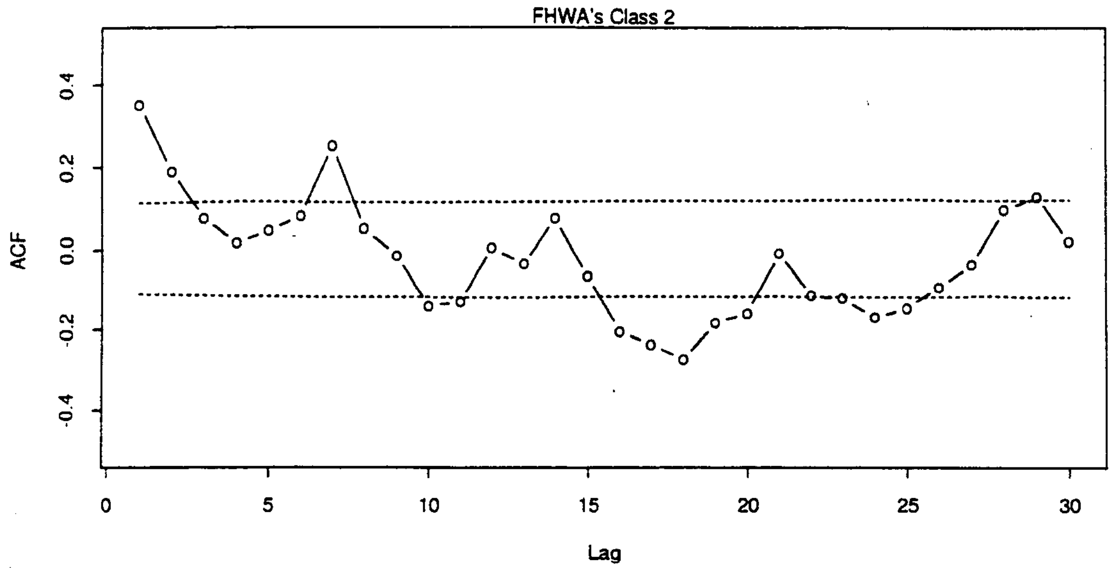


Figure 2.3 Standardized Residuals Plot for Poisson Model using Site 1023 (92)



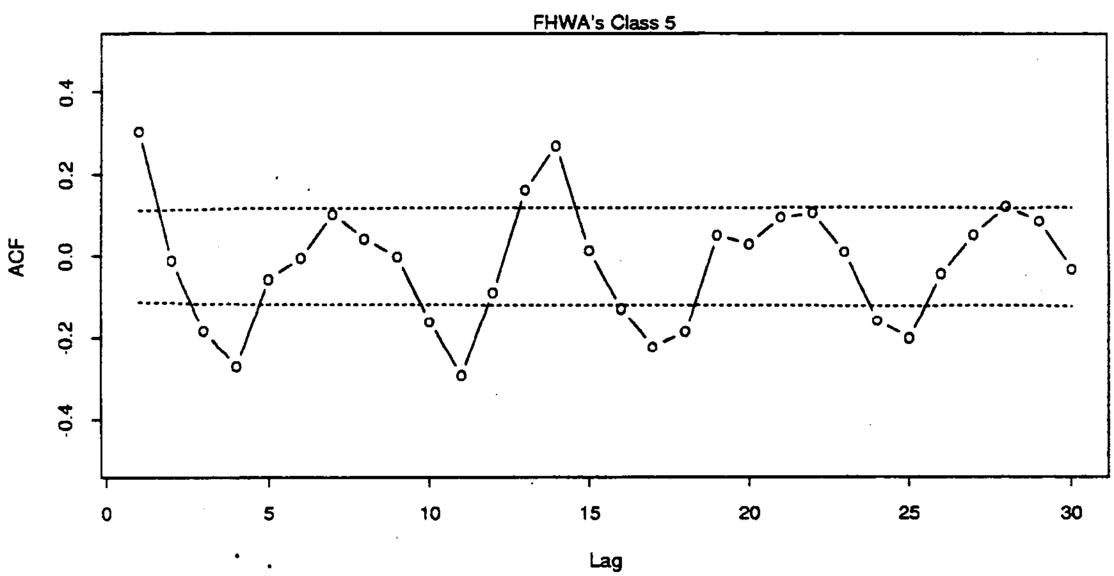


Figure 2.4 Autocorrelation Functions Plot of Poisson Model Residuals for Site 1023 (92)

of the regression coefficients were computed using an Iteratively Reweighted Least Squares (IRWLS) algorithm [16]. Chi-square test statistics were computed between the expected and the observed counts to evaluate the fit [17]. The statistics for WIM sites in 1992 were listed in Table 2.2. We found that although the Negative Binomial model reproduced the frequency distributions, an autocorrelation function plot made to test the independence of the residuals still showed serial correlation. Figure 2.5--Figure 2.7 showed the Negative Binomial model fit for site 1023 in 1992.

Table 2.2 χ^2 Statistics for Negative Binomial Model

χ^2 Statistics (Degree of Freedom)		
Site	FHWA Class 2	FHWA Class 5
1019	12.21(6)	12.24* (4)
1023	7.68 (7)	12.97 (6)
1029	11.84 (6)	24.65* (5)
1085	11.28* (4)	34.78* (7)
4033	2.24 (4)	16.45* (5)
4040	4.48 (5)	4.01 (3)
6251	2.10 (3)	44.34* (4)
9075	4.64 (3)	5.02 (3)

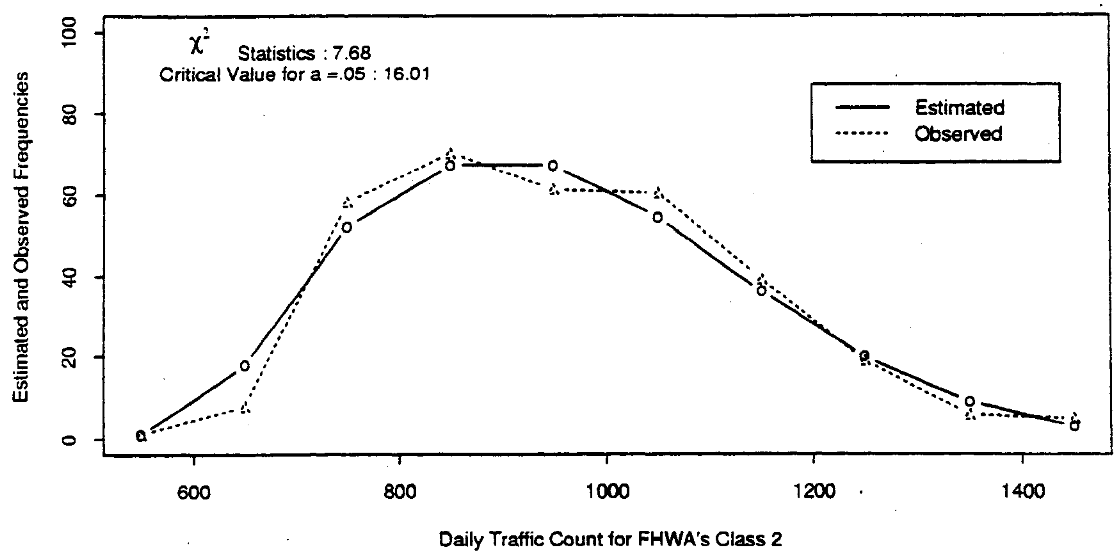
Note: * Reject the hypothesis at the 5% significant level

2.2.3 Lognormal Model

As mentioned before, the multiplicative model for traffic counts can also be implemented by assuming that daily classification traffic counts be Lognormal random variables, i.e, that the natural logarithm of the classification counts follows a linear regression model of the form

$$\log(z_t^{(k)}) = \mu^{(k)} + \sum_{i=1}^{12} \Delta_{t,i} m_i^{(k)} + \sum_{j=1}^{7} \delta_{t,j} w_j^{(k)} + \varepsilon_t^{(k)} \tag{2.2}$$

where



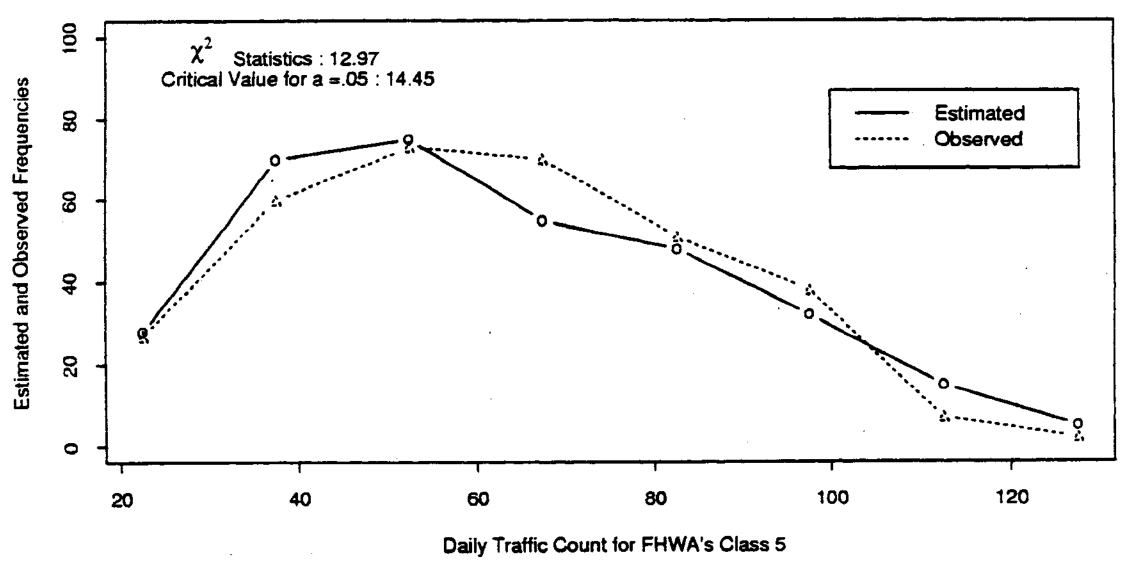


Figure 2.5 Estimated and Observed Frequencies for Negative Binomial Model using Site 1023 (92)

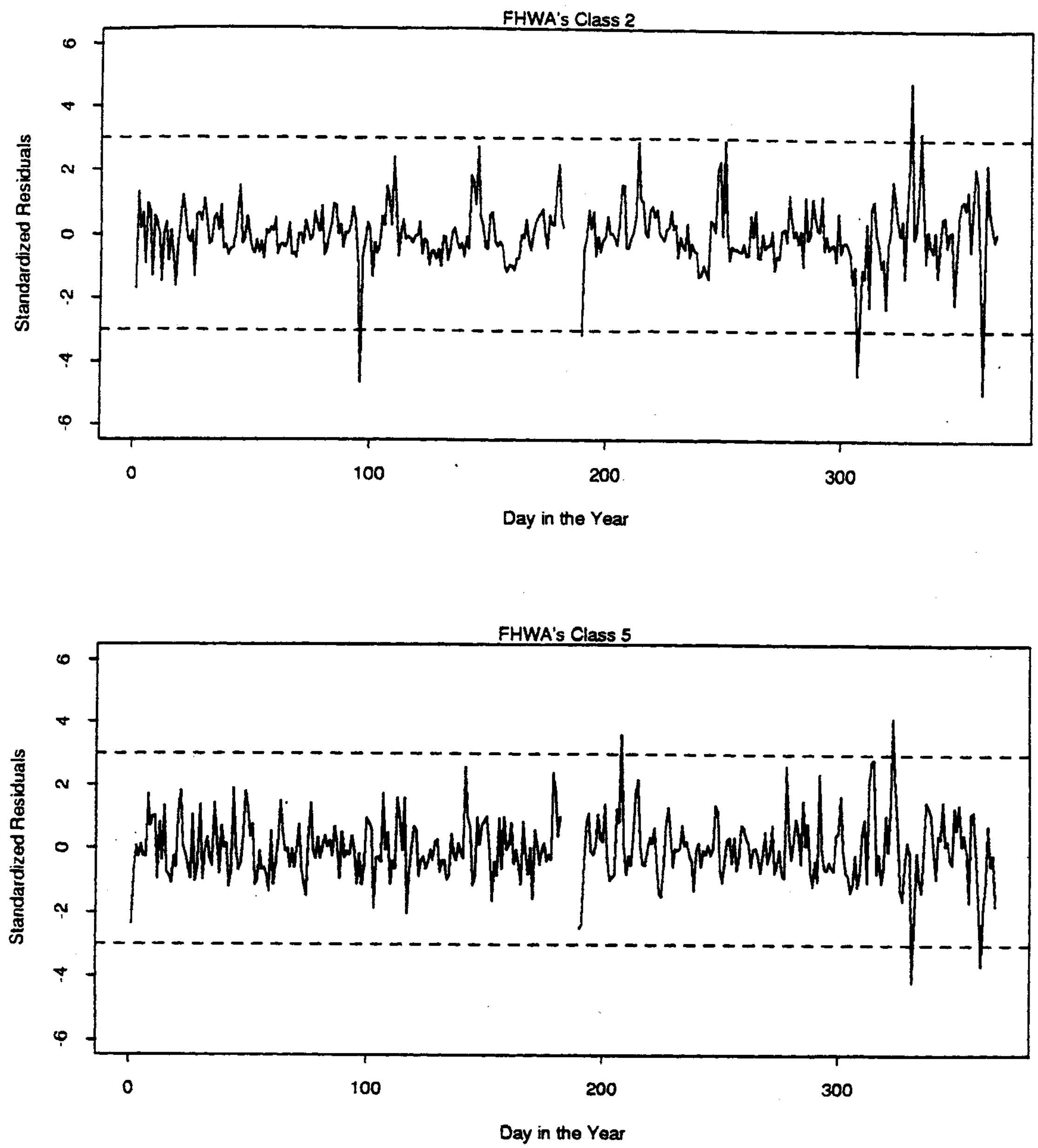


Figure 2.6 Standardized Residuals Plot for Negative Binomial Model using Site 1023 (92)

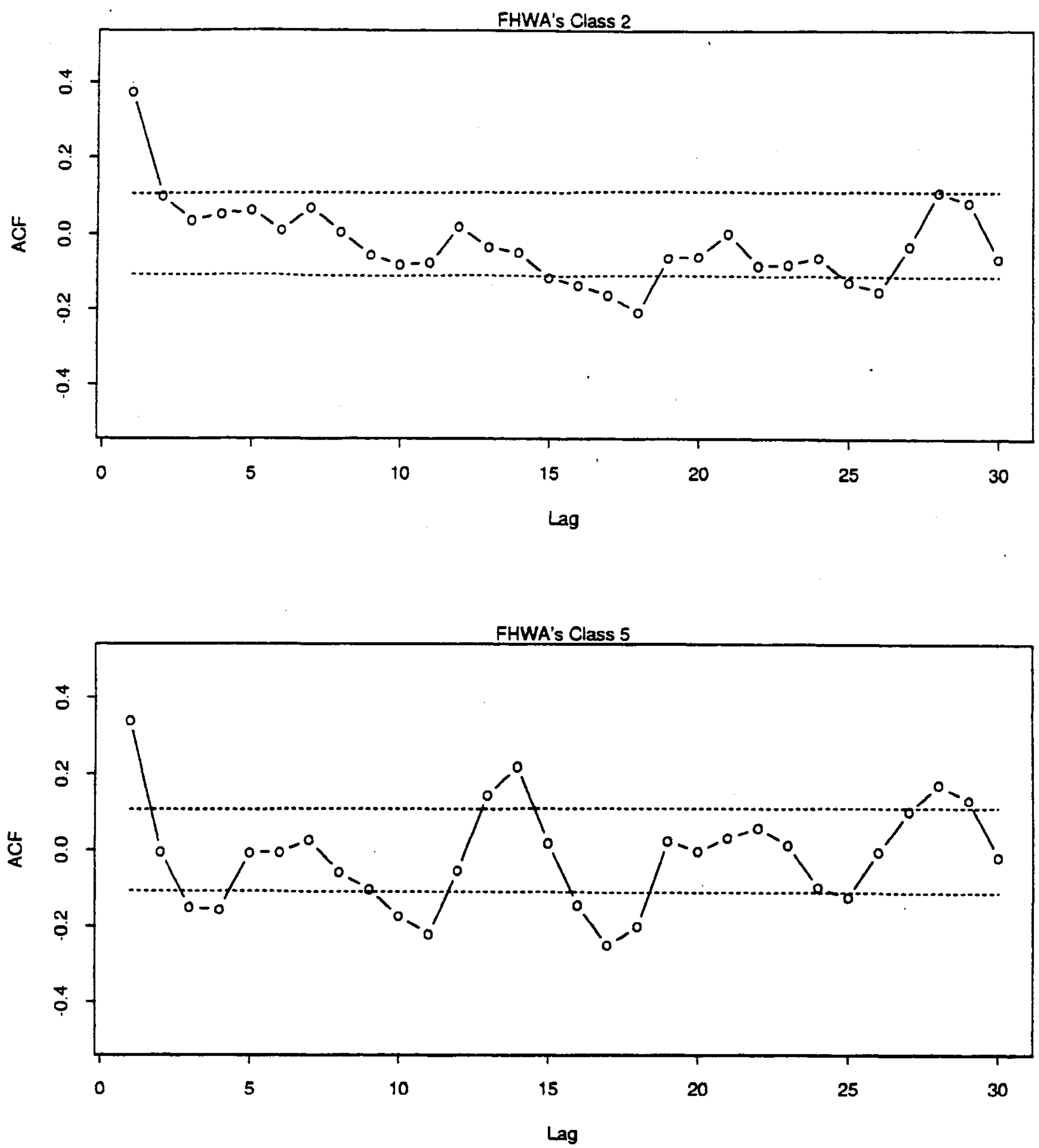


Figure 2.7 Autocorrelation Functions Plot for Negative Binomial Model using Site 1023 (92)

$\log_e(z_t^{(k)})$ = the natural logarithm of the traffic count on day t for count class k;

$\mu^{(k)}$ = expected log traffic count for count class k;

$\Delta_{t,i}$ = 1, if the count z_t was made during month i, i=1,...,12,

0, otherwise;

$m_i^{(k)} = \log_e(M_i^{(k)})$ = adjustment factor for month i, count class k;

$\delta_{t,j}$ = 1, if the count z_t was made on day-of-week j, j=1,...,7,

0, otherwise;

$w_j^{(k)} = \log_e(W_j^{(k)})$ = adjustment factor for day-of-week j, count class k;

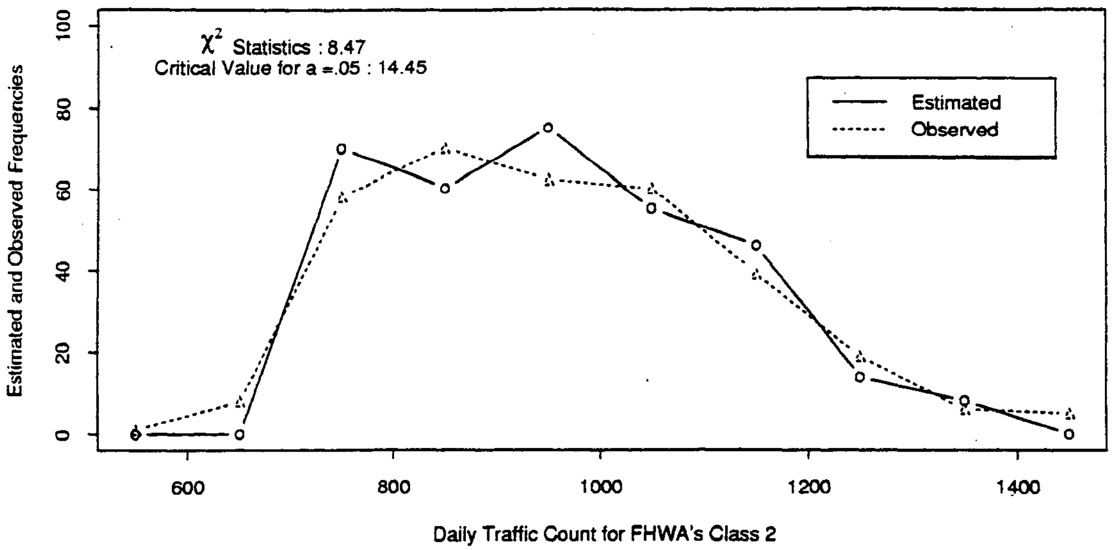
$\epsilon_t^{(k)}$ = random error (residual) for count class k.

We fit the Passenger Car and Single Unit Truck counts using the Lognormal model and found results are similar to those from the Negative Binomial model. Table 2.3 showed the comparable test statistics for the Lognormal model. The residuals were also calculated and the autocorrelation function plots were made to check the independence of the residuals. Again, serial correlation could be found among the residuals. Figure 2.8 and Figure 2.9 depict the Lognormal model fit for site 1023 in 1992.

Table 2.3 χ^2 Statistics for Lognormal Model

χ^2 Statistics (Degree of Freedom)		
Site	FHWA Class 2	FHWA Class 5
1019	12.02 (7)	18.34* (3)
1023	8.47 (6)	7.17 (6)
1029	4.88 (4)	12.79* (3)
1085	6.04 (3)	22.60* (6)
4033	5.63 (4)	39.50* (4)
4040	11.59* (3)	1.20 (3)
6251	1.45 (2)	5.99 (4)
9075	3.14 (2)	1.30 (2)

Note: * Reject the hypothesis at the 5% significant level



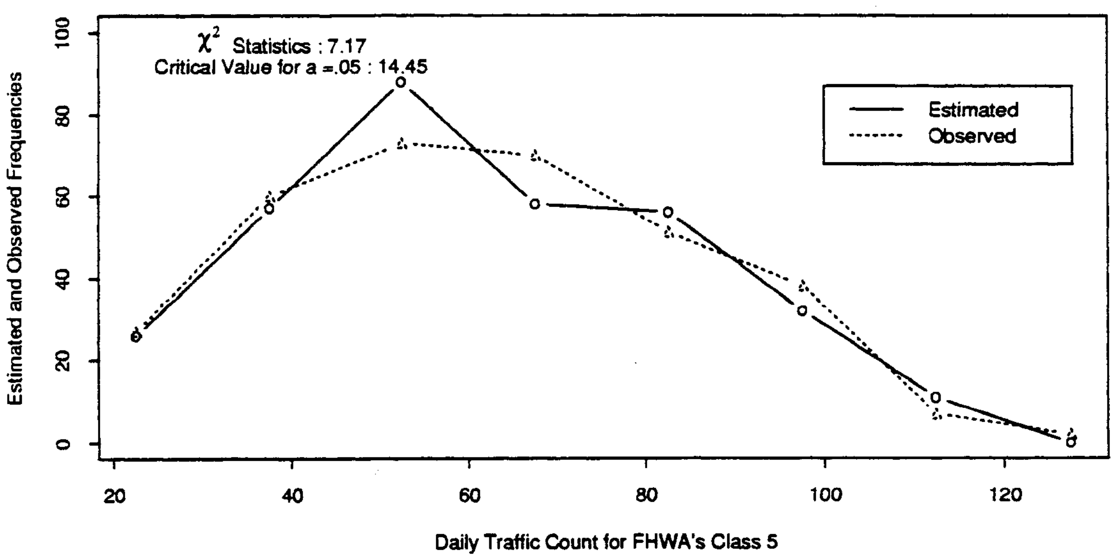
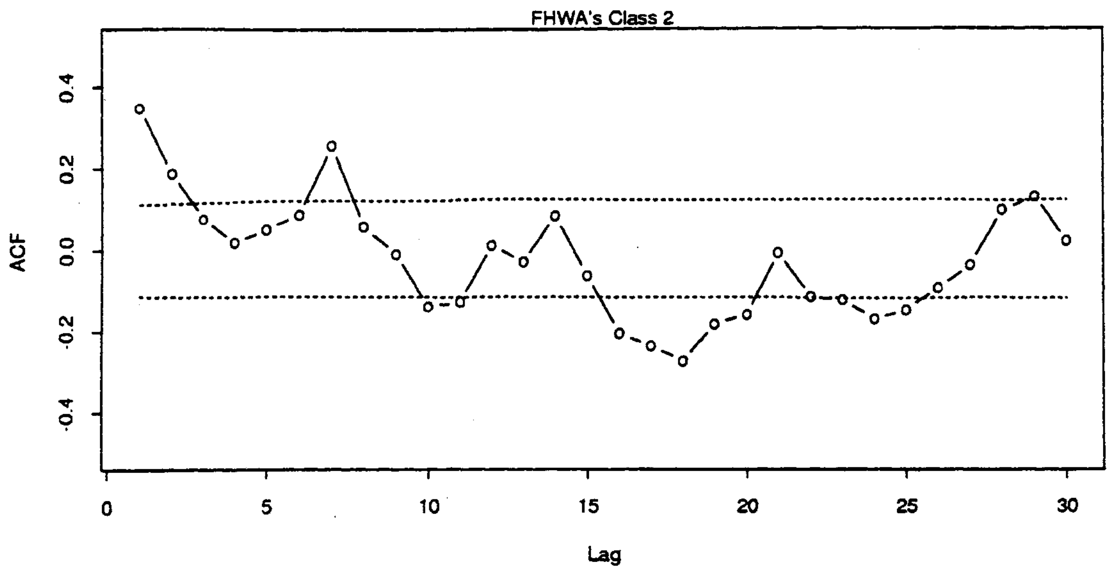


Figure 2.8 Estimated and Observed Frequencies for Lognormal Model using Site 1023 (92)



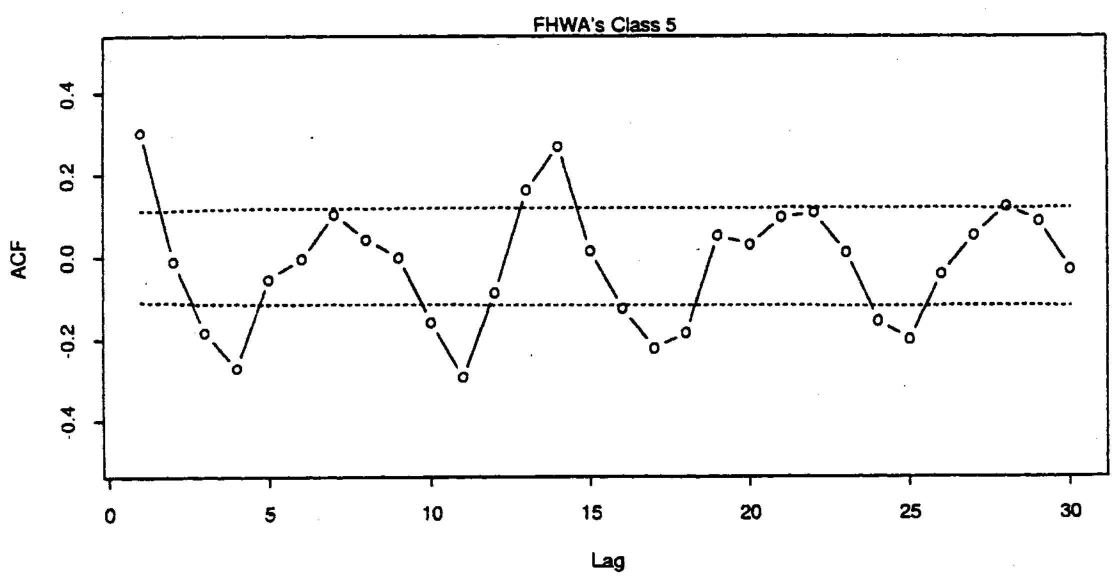


Figure 2.9 Autocorrelation Functions Plot for Lognormal Model using Site 1023 (92)

From this preliminary analysis, the following conclusions were drawn: First, the classification traffic counts tended to show greater day-to-day variability, especially for larger trucks, than would be expected if the counts were Poisson distributed; secondly, the Lognormal and the Negative Binomial models tended to produce fits of roughly equal quality; finally the residuals from each model tended to show serial correlation even after taking the monthly and day-of-week variation into account.

2.3 Lognormal Daily Classification Traffic Count Model

2.3.1 Aggregated Count Classes

Since the analysis of a Negative Binomial model with correlated data is rather complicated, while powerful methods have been developed for fitting time-series regression models with normal errors, the Lognormal model described above tends to be more attractive. However, one problem arises for the Lognormal model when we try to fit other vehicle classes using the Lognormal regression model. In FHWA's 13 classifications, some count classes such as the Multi-Trailer Trucks have 0 occurrence on certain days. In this case, the Lognormal count model will fail because the logarithm of 0 is not finite. For these data, one way that will allow us to keep the attractive features of the Lognormal model without incurring the problem described above is to use aggregated count classes. We felt this to be legitimate because research comparing aggregated traffic classifications and the 13-FHWA classification scheme showed that the aggregated count class scheme makes the seasonal factors for larger vehicle categories be more stable and thus more capable of predicting their volume [10]. A discussion of models that do allow for 0 counts is given in Appendix C. We thus aggregated the classification counts into three categories, and Table 2.4 shows how the aggregated count classes were constructed from the 13 FHWA count classes.

2.3.2 Lognormal Model for Multiple Counts

Let $\mathbf{z} = [\, \mathbf{z}_1, \ldots, \mathbf{z}_N \,]$ be a sample of classification counts collected in a highway site of interest,

Table 2.4 Aggregated Count Classes

Aggregated Count Classes		
Count Category	Name	FHWA Class*
Count Category 1	Passenger Cars / Pickups	Class 2 -- Class 3
Count Category 2	Single Unit Trucks / Buses	Class 4 -- Class 7
Count Category 3	Combination Trucks	Class 8 -- Class 13

Note: * Count Class 1 in the FHWA 13 class scheme is motorcycles, which were not of interest.

with $\mathbf{z}_t = (z^{(1)}, z^{(2)}, z^{(3)})'$, t = 1,..., N. Then the natural logarithm of the classification count on day t can be denoted as $\mathbf{y}_t = (y^{(1)}, y^{(2)}, y^{(3)})'$. If we denote $\beta = (\beta^{(1)}, \beta^{(2)}, \beta^{(3)})'$, where $\beta^{(k)} = (m_1^{(k)}, \ldots, m_{12}^{(k)}, w_1^{(k)}, \ldots, w_7^{(k)})'$, k= 1,..,3, be a column vector containing the monthly and day-of-week adjustment factors characteristic of class k, and denote $\boldsymbol{\mu} = (\mu^{(1)}, \mu^{(2)}, \mu^{(3)})'$ with $\mu^{(k)}$ be the expected log traffic count for each class, then the Lognormal model for multiple counts can be expressed as

$$\mathbf{y}_t = \mathbf{I}_3 \mu + \tilde{\mathbf{X}}_t \beta + \varepsilon_t \tag{2.3}$$

Here $\mathbf{I}_3$ denotes a 3 by 3 identity matrix and $\tilde{\mathbf{x}}_t$ is a matrix of dimension 3 by (3×19), with elements in row i and columns (i-1)×19+1 containing the value 0 or 1 depending on the month and day-of-week for the count on that day and the elements in all the other positions being 0. Let us further denote $\mathbf{y} = (\mathbf{y}_1, \ldots, \mathbf{y}_N)'$ and $\epsilon = (\epsilon_1, \ldots, \epsilon_N)'$ with $\mathbf{y}_t = (y^{(1)}, y^{(2)}, y^{(3)})'$, $\epsilon_t = (\epsilon^{(1)}, \epsilon^{(2)}, \epsilon^{(3)})'$, t=1,...,N, $\mathbf{X} = (\tilde{\mathbf{x}}_1, \ldots, \tilde{\mathbf{x}}_N)'$ with $\tilde{\mathbf{x}}_t$ be the matrix defined in (2.3) and denote $\mathbf{1}_{3N\times3} = (\mathbf{I}_3, \ldots, \mathbf{I}_3)'$ be a matrix of dimension (3xN) by 3, then we can write (2.3) in a vector form as,

$$\mathbf{y} = \mathbf{1}_{3N\times3}\mu + \mathbf{X}\beta + \varepsilon \tag{2.4}$$

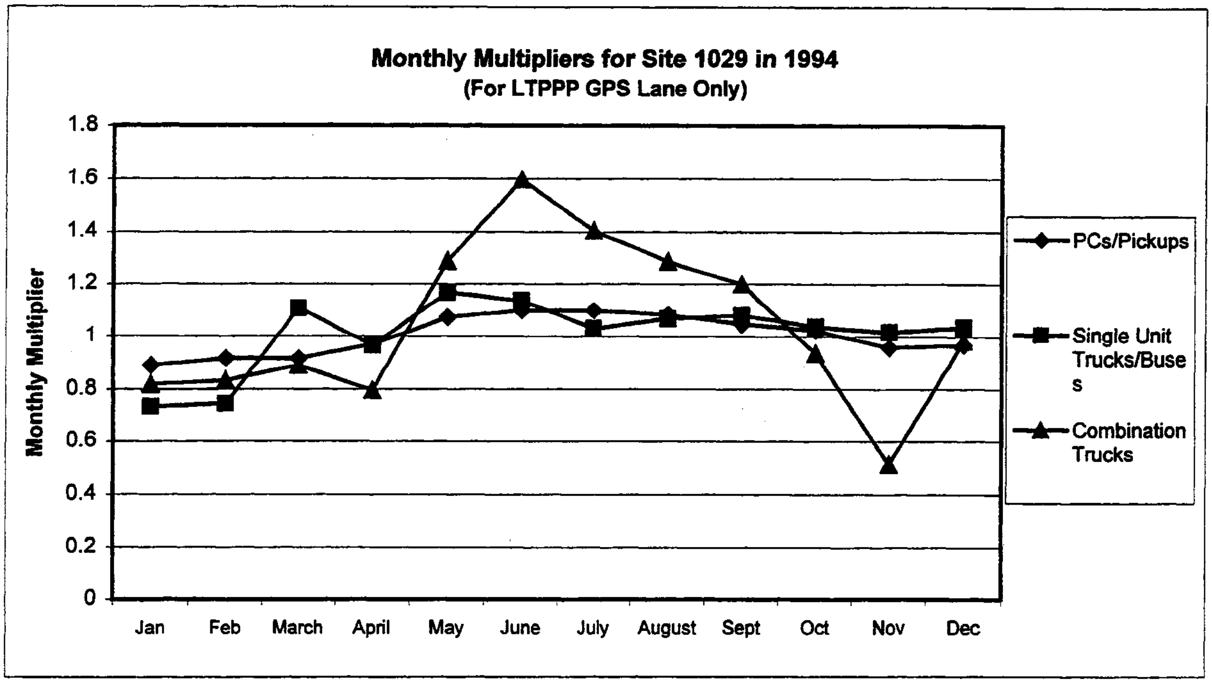
This model will be fit to data using a two stage process. The first stage is to estimate the set of parameters containing the logarithm of the classification mean daily traffic (μ) and the monthly and day-of-week adjustment factors (m_i and w_j in β) described in equation (2.2) for each aggregated category. The second stage consists of fitting time-series models to the regression residuals ϵ obtained from the first stage, and then testing the goodness-of-fit of the model.

2.3.3 First Stage Model Fitting

We use least squares regression to estimate the mean μ, and the monthly and day-of-week adjustment factors β for each of the 24 WIM data sets. Exponentiating the estimates of $m_i^{(k)}$ and $w_j^{(k)}$ in β thus produces estimates of monthly multipliers $M_i^{(k)}$ and day-of-week multipliers $W_j^{(k)}$ defined in the multiplicative model in (2.1). Obviously, if a site lacks data for an entire month, that monthly term cannot be estimated, and so estimates from such a site should not be used to adjust short counts. Here are some results from the first stage fitting.

First, the seasonal and day-of-week trend for different count classes were quite different, especially for the day-of-week trend. Figure 2.10 shows these monthly and day-of-week multipliers for different count categories for WIM site 1029 in the year of 1994. Comparatively, the monthly and weekly trend for the first category is relatively stable, while for the larger trucks, greater variations could be clearly found, with the monthly multipliers ranging from 0.5 to 1.6 and day-of-week multipliers ranging from 0.3 to 1.6 for the combination trucks. The second category is intermediate. Another pronounced characteristic is that the weekly trend for larger trucks is an upside down "bathtub curve" because of low occurrences of trucks during weekends. Thus, using the seasonal adjustment factors for the passenger cars to adjust the short period truck traffic would, in most cases, result in estimation errors.

Secondly, we have investigated the stability of the adjustment factors over the years. These can be seen in Figure 2.11 and Figure 2.12 which depict the monthly and day-of-week multipliers for site 1023 in the year of 1992, 1993 and 1994. The factors tended to be stable over the three years.



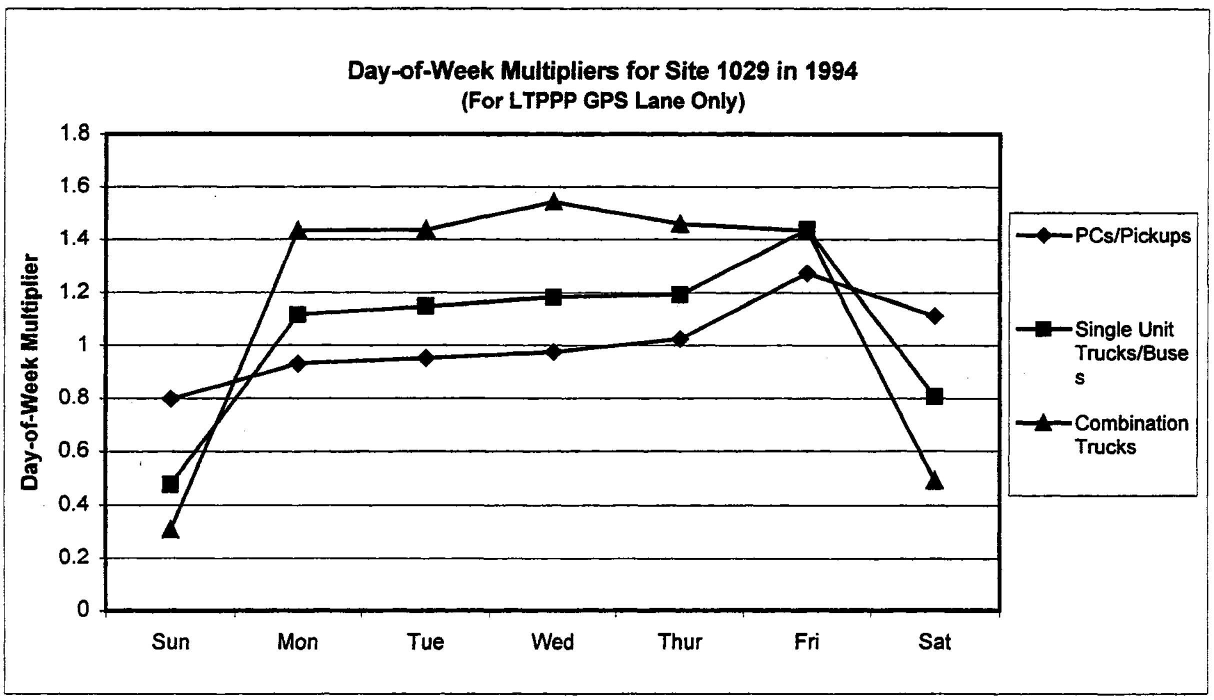
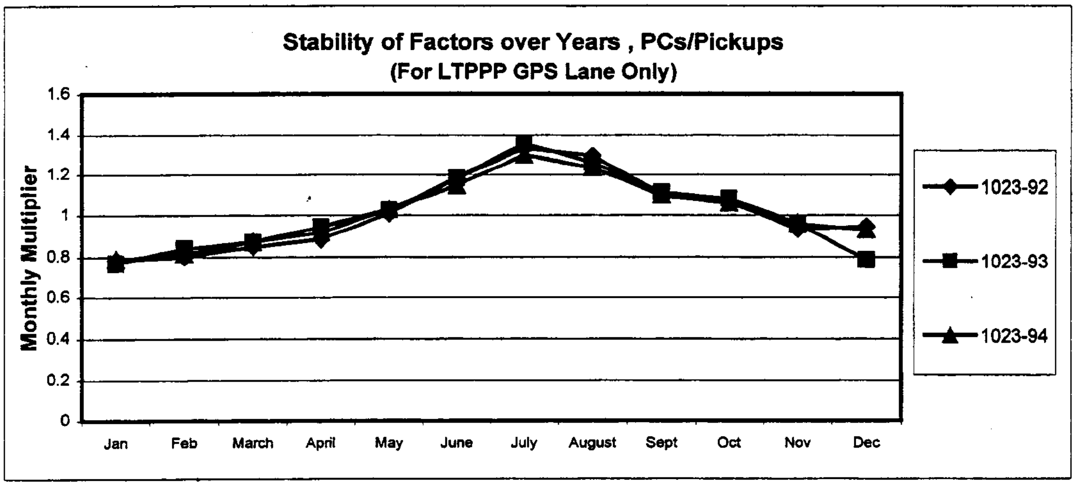
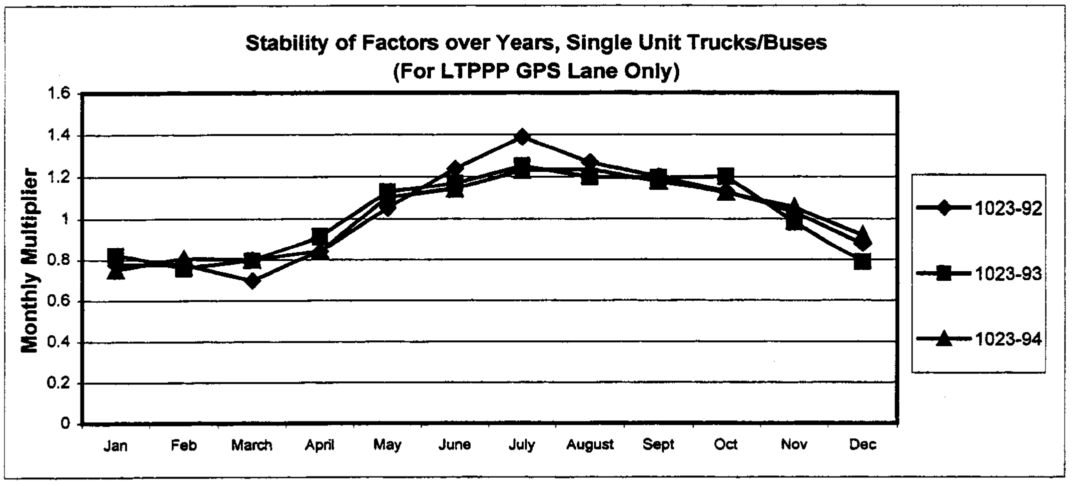


Figure 2.10 Estimated Monthly and Day-of-Week Multipliers using Site 1029 (94)





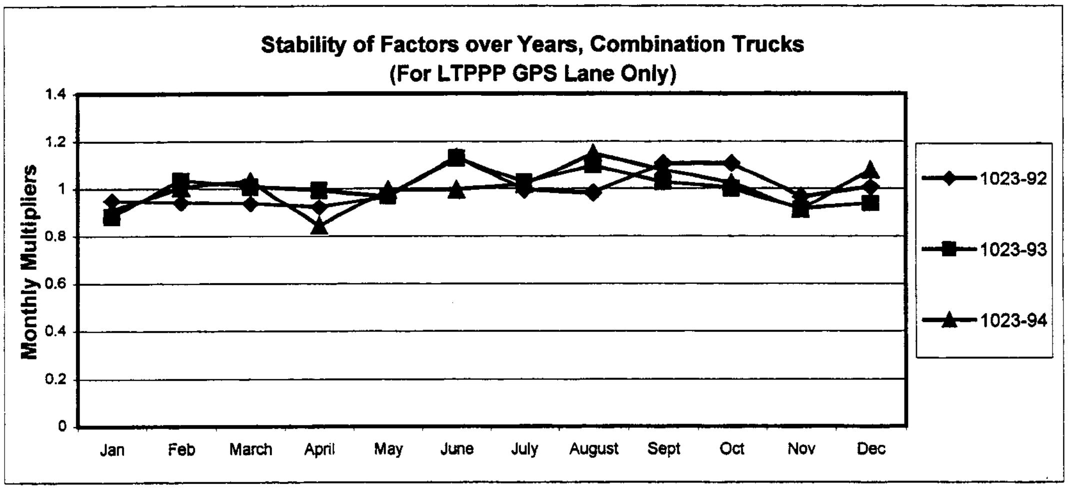
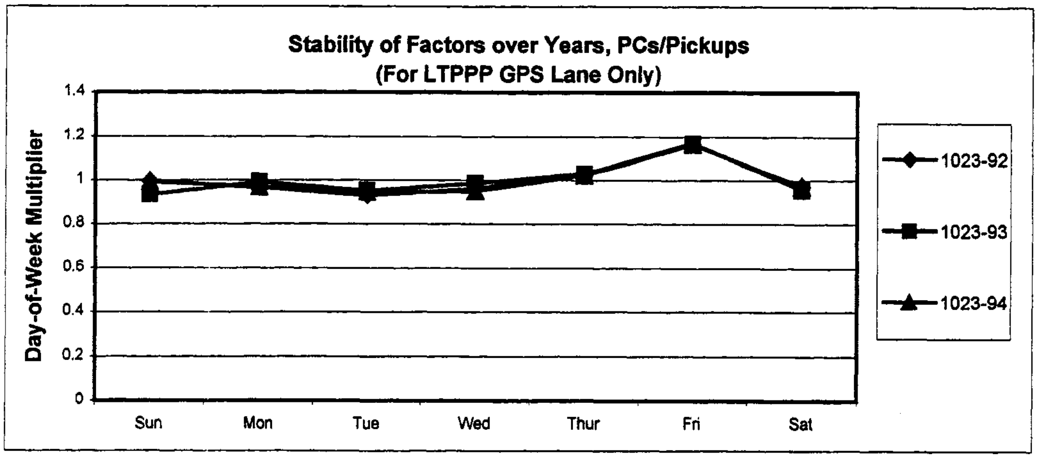
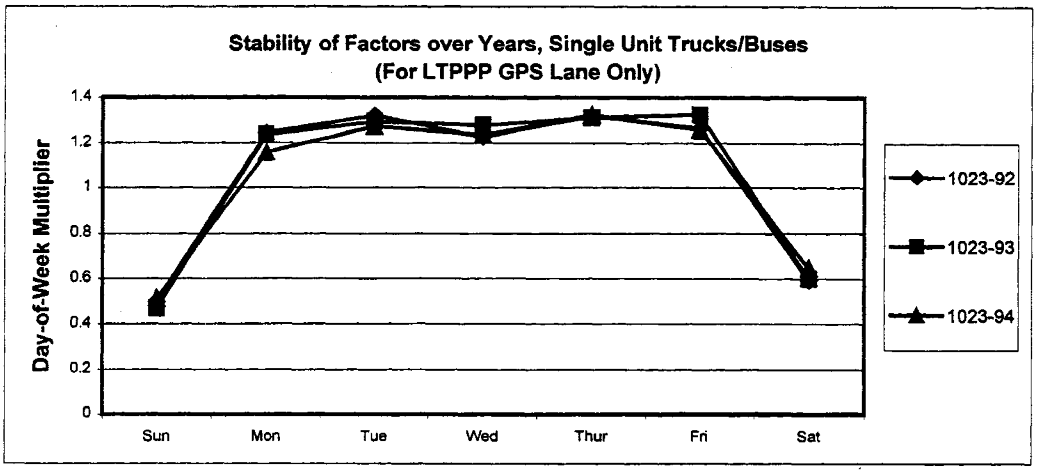


Figure 2.11 Stability of Monthly Multipliers Over Three Years using Site 1023





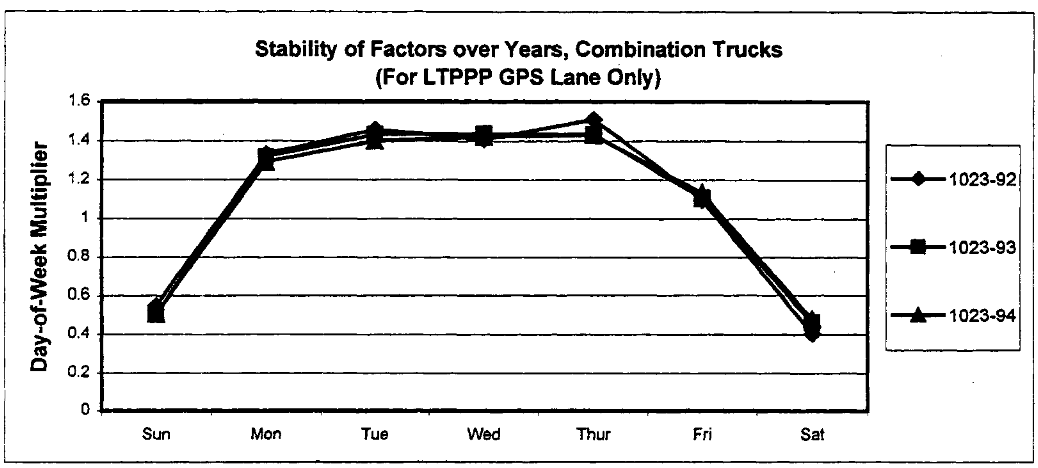


Figure 2.12 Stability of Day-of-Week Multipliers Over Three Years using Site 1023

2.3.4 Second Stage Modeling and Goodness-of-Fit

In the second stage, the main objective is to determine a plausible form for a time series model of the first stage residuals ϵ, and to verify that the second stage residuals are distributed normally. The standard attack on the question of whether the residuals $\epsilon_i^{(k)}$ are independent is to check for serial correlation by correlating the series $\{\epsilon_i^{(k)}\}$ with the series $\{\epsilon_{i-l}^{(k)}\}$, where l is the size of the lag between observations being correlated, i.e, first letting $l=1$ to compute the correlation coefficient of observations separated by one day, then letting $l=2$ to compute the correlation coefficient of observations separated by two days, and till reaching the maximum lag m specified. With a lag of l one has a set of n-l pairs of observations to be used in calculating the autocorrelation function (ACF) coefficients. In time series analysis, the estimated autocorrelation function computed from the residuals provides the key tool for identifying the patterns of temporal dependency [18]. An ACF plot provides a graphical display of the estimated correlations between observations separated by lag, and thus can be used to identify the pattern of the underlying time series model. Therefore, the autocorrelation functions with the maximum lag $m=30$ were computed for the first stage residuals $\epsilon^{(k)}$ for all the 24 classification traffic data sets, using Dunsmuir and Robinson's [19] procedure which allows for missing data. Figure 2.13 shows the estimated ACF for site 1023 in the year of 1994. The horizontal dotted lines provide an approximate 95% confidence interval for the autocorrelation estimate at each lag. We can see that for the first count category, the autocorrelation estimate at lag 1 falls outside the strip defined by the upper dotted line, and then the estimates damped down for the latter lags. Similar patterns can be seen for the second count category. For the third category, significant correlation can be found at lag 1 and then the coefficients damped down slowly with some falling outside of the negative dotted line. From this pattern, a first-order autoregressive (AR(1)) effect can be conjectured.

We then fit a multivariate autoregressive model to the first stage residuals, which is of the form,

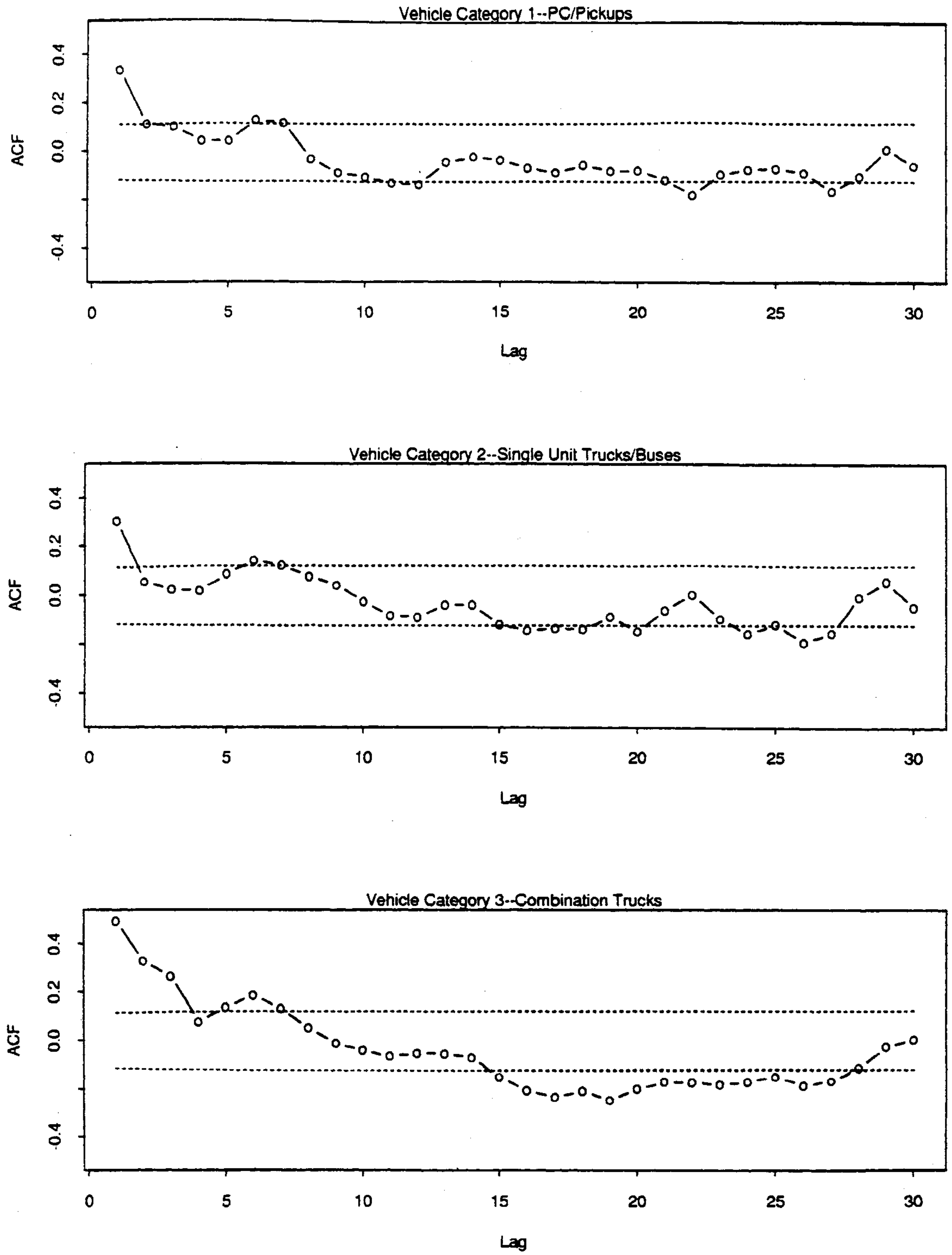


Figure 2.13 Autocorrelation Functions Plot for Stage 1 Residuals using Site 1023 (94)

$$\begin{bmatrix} \varepsilon_t^{(1)} \\ \varepsilon_t^{(2)} \\ \varepsilon_t^{(3)} \end{bmatrix} = \begin{bmatrix} \phi_{1,1} & \phi_{1,2} & \phi_{1,3} \\ \phi_{2,1} & \phi_{2,2} & \phi_{2,3} \\ \phi_{3,1} & \phi_{3,2} & \phi_{3,3} \end{bmatrix} \begin{bmatrix} \varepsilon_{t-1}^{(1)} \\ \varepsilon_{t-1}^{(2)} \\ \varepsilon_{t-1}^{(3)} \end{bmatrix} + \begin{bmatrix} \alpha_t^{(1)} \\ \alpha_t^{(2)} \\ \alpha_t^{(3)} \end{bmatrix} \tag{2.5}$$

or in a vector form,

$$\epsilon_t = \Phi\epsilon_{t-1} + \alpha_t$$

where

$\epsilon_t^{(k)}$ = first stage residuals for count category k , k=1,...,3 at time t,

$\epsilon_{t-1}^{(k)}$ = first stage residuals for count category k , k=1,...,3 at time t-1,

ϕ = 3x3 matrix characterizing the multivaritate autoregressive AR (1) parameters,

$\alpha_t^{(k)}$ = second stage residuals for count category k , k=1,...,3 at time t.

We first obtain the estimates of the multivariate AR (1) parameters using the least squares regression method and then compute the second stage residuals via

$$\alpha_t = \epsilon_t - \Phi\epsilon_{t-1} \tag{2.6}$$

where the α_t are normally distributed with means of 0 and a covariance matrix Σ, i.e, $\alpha_t \sim N(0, \Sigma)$. The diagonal term of Σ is the variance of each sequence representing each category and the element in row s and column t, $\Sigma_{s,t}$ is the product of the correlation coefficient for α_s and α_t and the standard deviations of α_s and α_t. If the AR(1) model adequately describes the correlation pattern, the three sequences of second stage residuals should (1) show no significant autocorrelations and (2) pass a goodness-of-fit test for normally distributed data.

To test the first hypothesis, the second stage residuals were computed via (2.6) and the autocorrelation function for the second stage residuals were obtained along with the critical values for testing the hypothesis that no estimated autocorrelation coefficient is significantly different from zero [20]. Figure 2.14 displays the results of these computations and it can be seen that the AR(1) process did account for the dependency shown in Figure 2.13.

To test the normality of these second stage residuals, a normal probability plot of the standardized second stage residuals was prepared, for each series, as well as the correlation coefficient between the residuals and their normal scores. Figure 2.15--Figure 2.17 display these results for the 3 classes for site 1023 in year 1994. If the data are normally distributed, the scatter plot in Figure 2.15--Figure 2.17 should be approximately linear. Figure 2.15 showed that the correlation coefficient for the Passenger Cars/Pickups category, $R=0.9748$, to be significantly different from 1.0 ($p<0.01$), and we should reject the hypothesis that these second stage residuals are normally distributed. However the inspection of the normality plot shows that the deviation from normality are caused by a few extreme values at the tails of the distribution. If we consider the most extreme values as atypical outliers and remove them from the data, we could obtain a better goodness of fit results shown in Figure 2.18. A similar situation was found for the third vehicle category, and the normal plot after removing several outliers is shown in Figure 2.19. It was felt reasonable to remove outliers because atypical counts can appear due to weather, special events or device malfunctioning.

Similar testing was done for all the WIM sites and years, and the results shown here turned out to be very representative. The autocorrelation functions of each class for the first stage residuals showed patterns similar with that in Figure 2.13. These autocorrelations were essentially removed after fitting a multivariate AR model in (2.5). The second stage standardized residuals for the Passenger Cars/Pickups class passed a normality test 40 percent of time, without the consideration of outliers. The second stage residuals for the Single Unit Truck class passed a normality test 30 percent of time and the Combination Trucks class passed 10 percent of time. However, if we are willing to make the plausible assumption that there do appear extreme count values, defined as those having standardized residuals with absolute values greater than 2.5~3.0, due to some undetected atypical events, then after removal of a small (i.e, 1-17) number of extreme values, almost all second stage residuals from three classes passed the normality test. Table 2.5

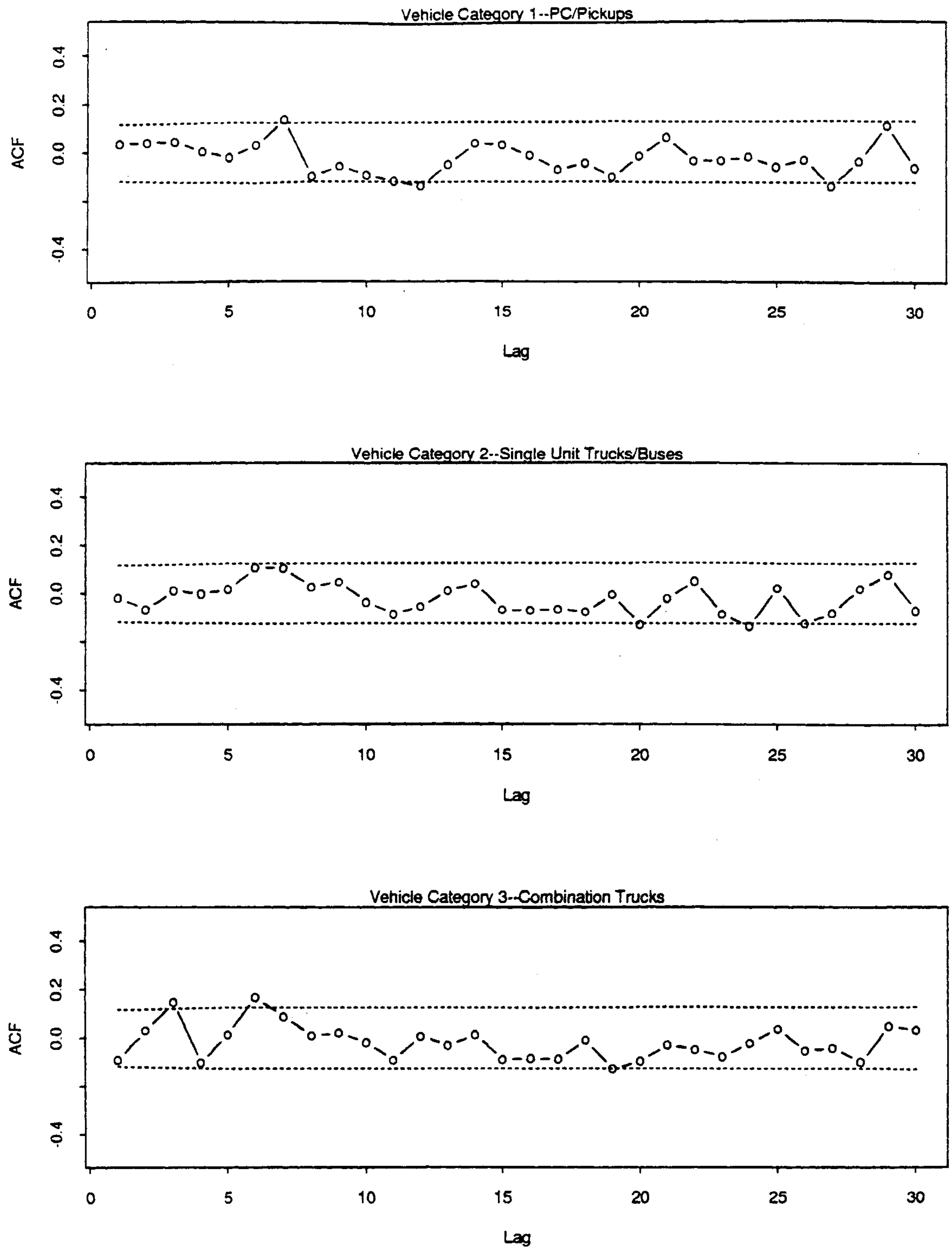


Figure 2.14 Autocorrelation Functions Plot of Stage 2 Residuals using Site 1023 (94)

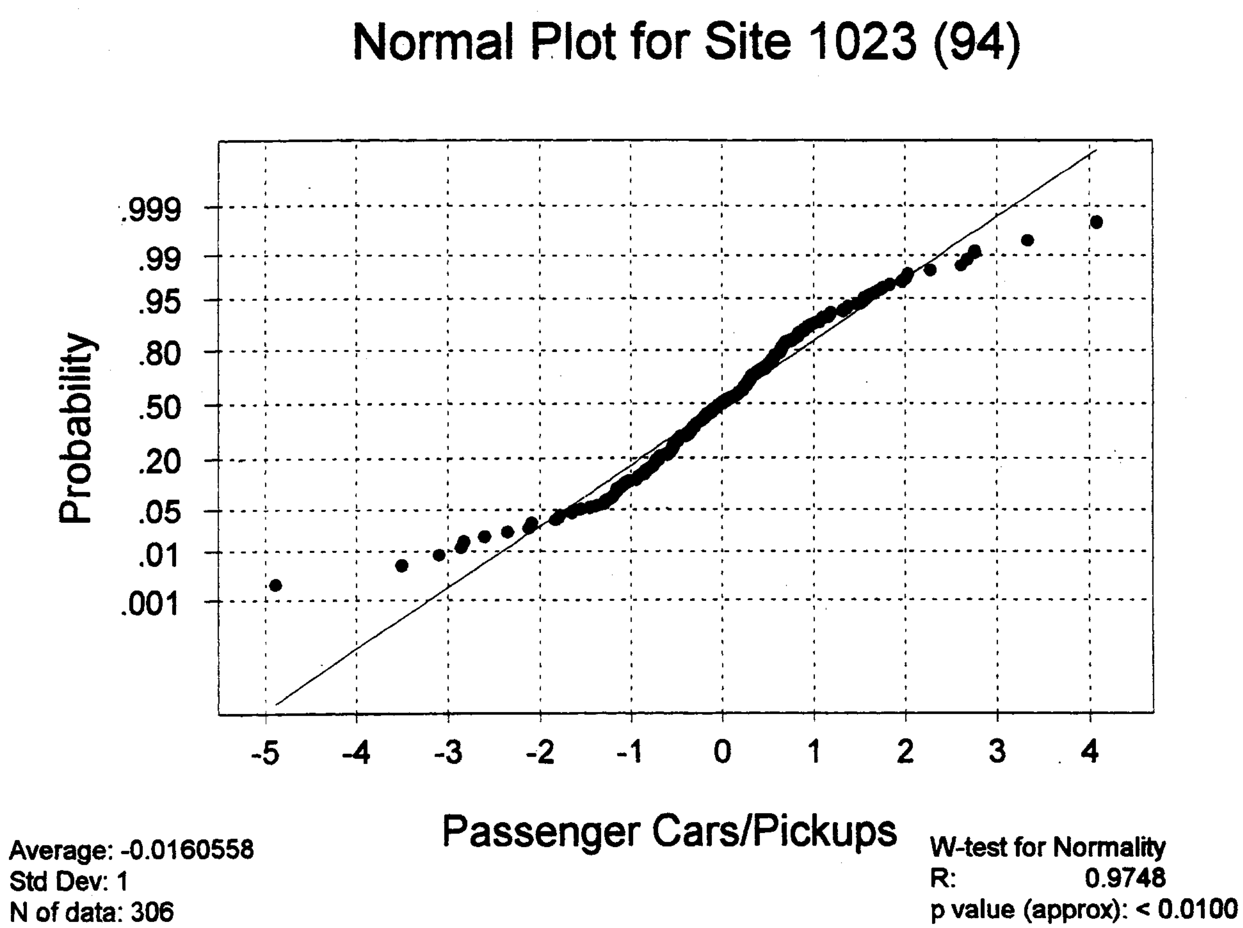


Figure 2.15 Normality Test Plot of Stage 2 Residuals for Count Category 1
using Site 1023 (94) No Outliers Removed

Normal Plot for Site 1023(94)

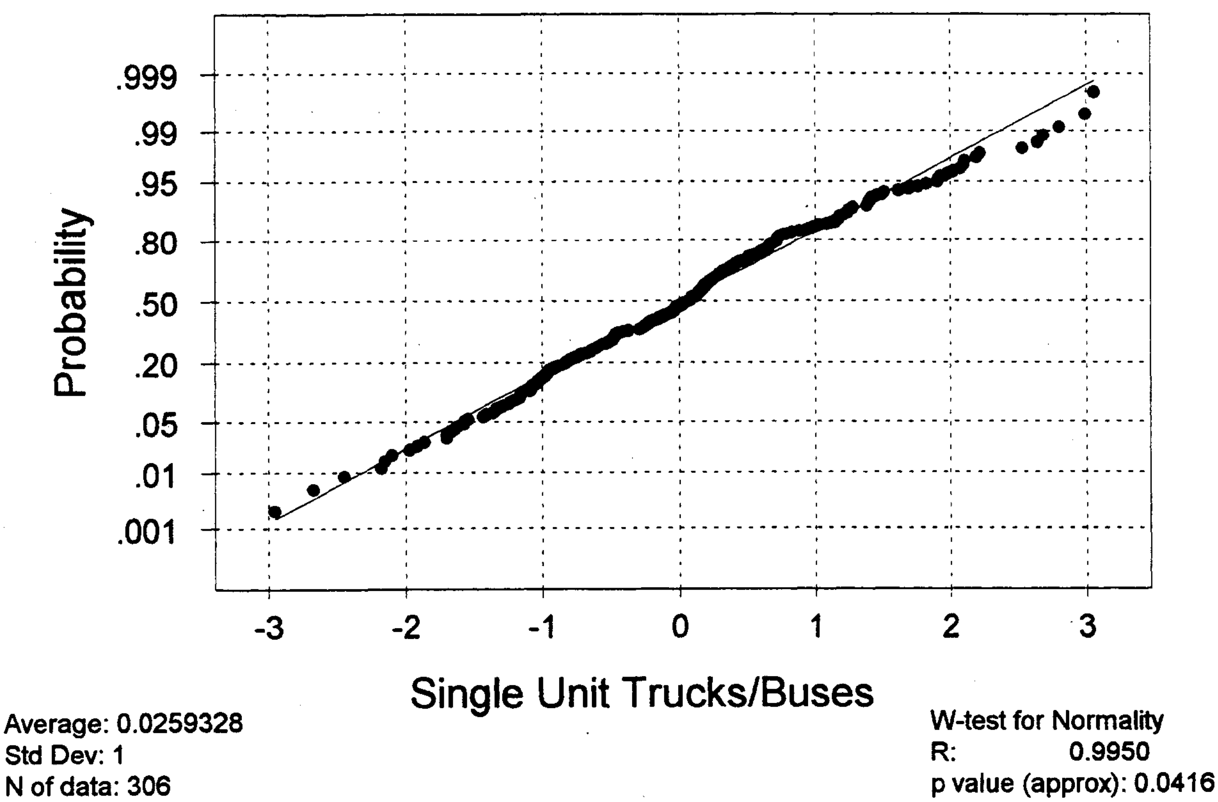


Figure 2.16 Normality Test Plot of Stage 2 Residuals for Count Category 2 using Site 1023 (94) No Outliers Removed

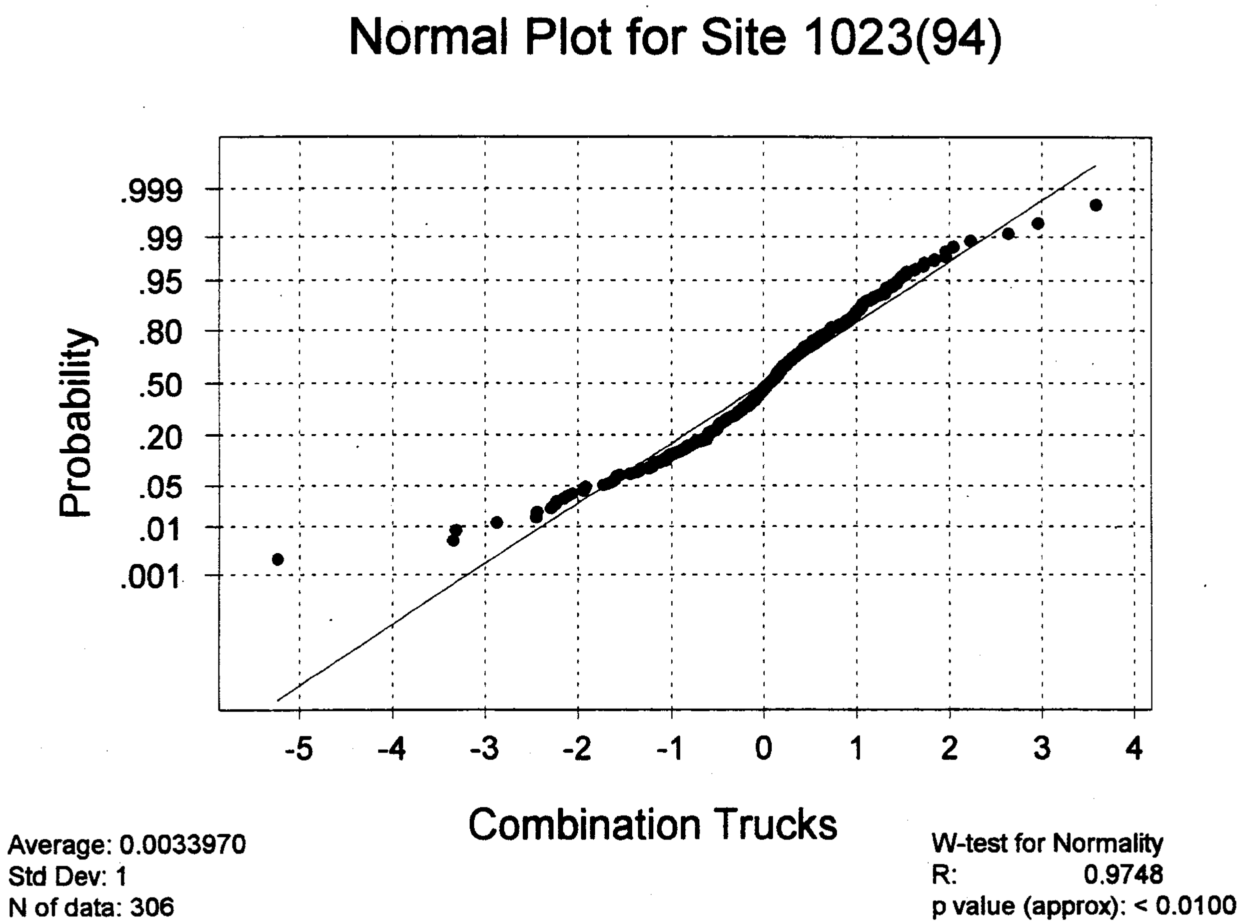


Figure 2.17 Normality Test Plot of Stage 2 Residuals for Count Category 3 using Site 1023 (94) No Outliers Removed

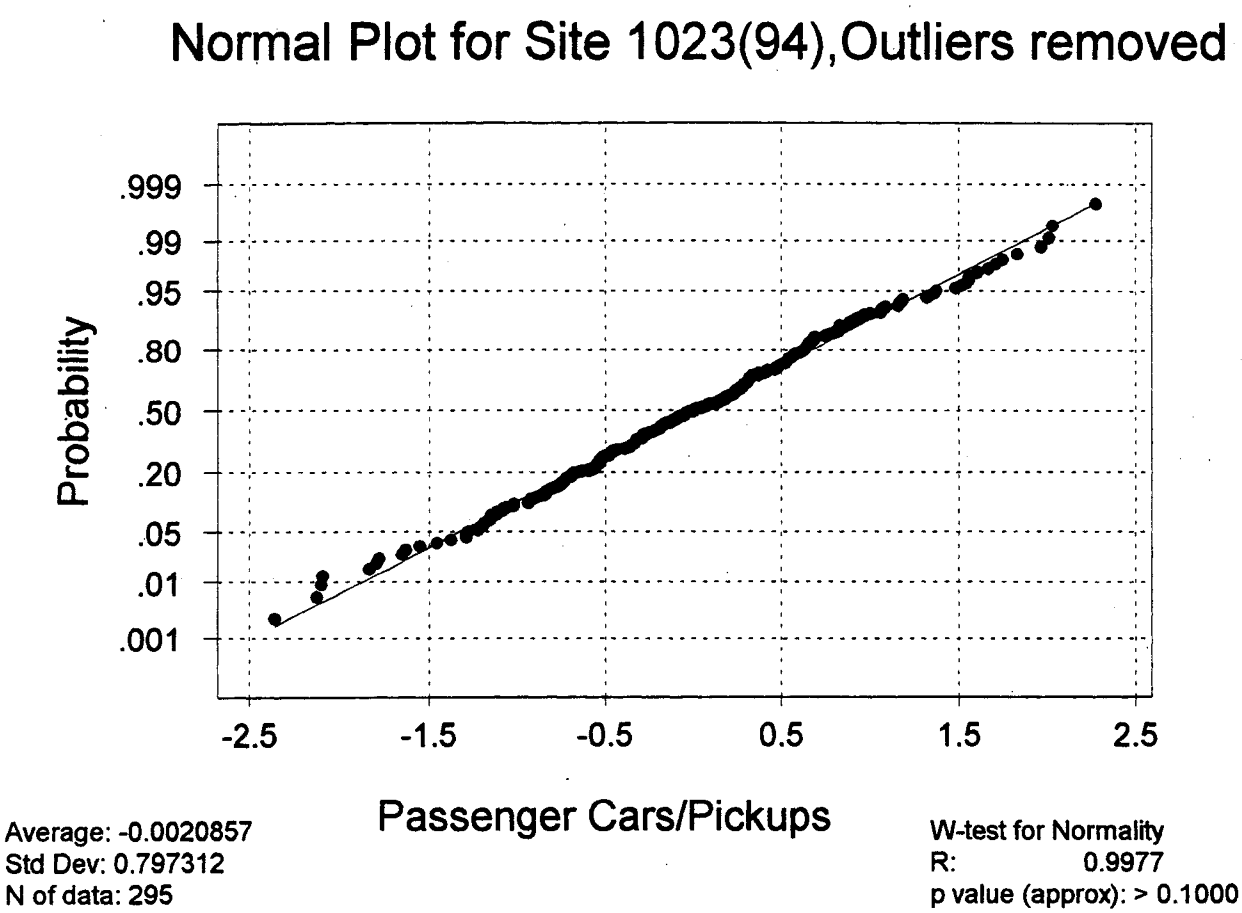


Figure 2.18 Normality Test Plot of Stage 2 Residuals for Count Category 1 using Site 1023 (94) Outliers Removed

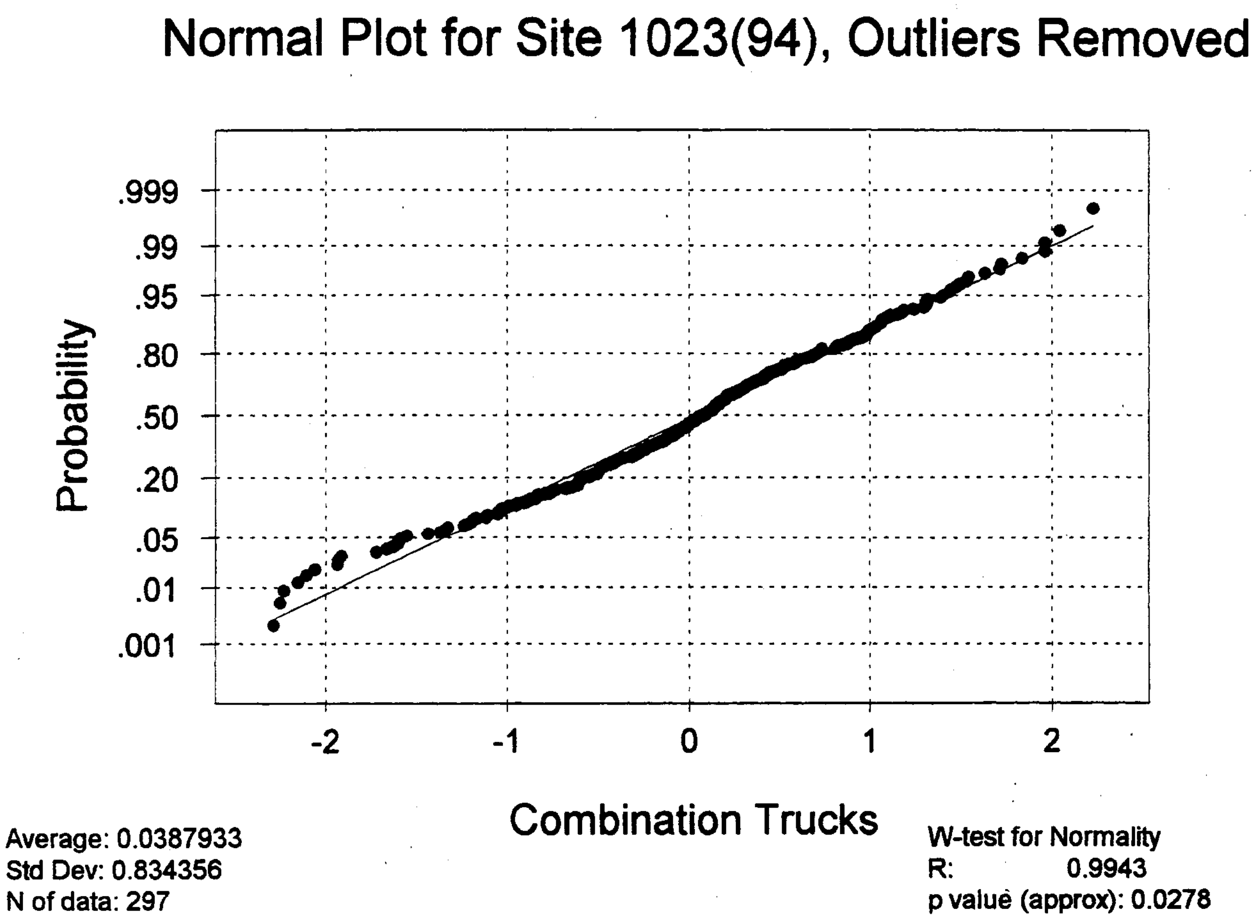


Figure 2.19 Normality Test Plot of Stage 2 Residuals for Count Category 3 using Site 1023 (94) Outliers Removed

summarizes the normal correlation test statistics for all the sites available for the study. Therefore, to fit the daily, non-holiday, one lane, one directional traffic classification counts in Minnesota, the Lognormal regression model given in (2.4), with residuals satisfying a multivariate AR(1) model of the form (2.5) is a defensible choice of statistical model.

2.4 Conclusion

Through the model fitting process, we have provided fairly strong evidence that a Lognormal regression model describes the statistical properties of daily classification traffic counts and thus provides the likelihood function. The connection with regression methods and time-series analysis means that a wide range of powerful statistical tools can be brought to bear on problems of traffic classification estimation. In the following chapters, Bayesian estimation theory will be applied to the Lognormal likelihood function to estimate the mean daily classification traffic.

Table 2.5 Normal Correlation Test Statistics for Lognormal Model with AR(1) Time series Model

Normal Correlation Test Statistics (Number of Outliers Removed)				
Year	Site	Category 1	Category 2	Category 3
1992	1019	0.9952 (14)	0.9964 (14)	0.9961 (14)
	1023	0.9967 (12)	0.9972 (9)	0.9972 (11)
	1029	0.9944 (9)	0.9976 (10)	0.9976 (12)
	1085	0.9971 (12)	0.9943 (15)	0.9981 (14)
	4033	0.9941 (1)	0.9983 (3)	0.9972 (4)
	4040	0.9961 (12)	0.9979 (15)	0.9979 (13)
	6251	0.9955 (9)	0.9976 (9)	0.9941 (13)
	9075	0.9981 (8)	0.9986 (7)	0.9984 (8)
1993	1023	0.9974 (14)	0.9964 (9)	0.9966 (17)
	4033	0.9968 (10)	0.9991 (8)	0.9972 (10)
	4037	0.9948 (6)	0.9979 (10)	0.9987 (4)
	4040	0.9903*	0.9938 (10)	0.9964 (11)
	6251	0.9972 (6)	0.9976 (15)	0.9985 (15)
	9075	0.9971 (9)	0.9986 (15)	0.9987 (11)
1994	1019	0.9980 (14)	0.9982 (7)	0.9978 (14)
	1023	0.9977 (11)	0.9975 (8)	0.9943 (9)
	1029	0.9974 (15)	0.9944 (9)	0.9941 (15)
	4033	0.9965 (13)	0.9887*	0.9938 (10)
	4037	0.9930*	0.9969 (6)	0.9811*
	4055	0.9985 (10)	0.9972 (15)	0.9907*
	6251	0.9984 (5)	0.9987 (11)	0.9970 (13)
1995	1019	0.9963 (13)	0.9979 (12)	0.9987 (10)
	1085	0.9950 (9)	0.9980 (10)	0.9981 (10)
	9075	0.9940 (12)	0.9982 (13)	0.9943 (6)

Note: * Sites that haven't passed the normality test after removal of 17 outliers

CHAPTER 3

BAYESIAN ASSIGNMENT TO FACTOR GROUPS

As stated in Chapter 1, because of the seasonal and day-of-week variations in daily classification traffic, simple averages of short-count data will produce biased estimates of mean daily traffic by vehicle types. The general method for estimating mean daily traffic involves two steps, first estimating the monthly and day-of-week adjustment factors for a representative set of permanent counters and the second matching the short count site to a permanent counter and then applying appropriate monthly and day-of-week factors to adjust the short-count data. The statistical methods developed in Chapter 2 can be used to estimate the monthly and day-of-week factors. The more difficult and also critical part of the whole process involves assigning short count sites to the appropriate factor group, since an incorrect assignment could lead to large estimation errors. In this chapter, Bayesian inference is addressed and then applied to the problem of assigning short-count sites to factor groups. Specifically, a Bayesian method is applied to the multivariate lognormal model of daily classification traffic counts to obtain the probability a short count site has adjustment terms similar to each of the WIM factor groups.

3.1 Bayesian Inference

Bayesian inference is a statistical approach in which all forms of uncertainty are expressed in terms of probability. A Bayesian approach starts with the formulation of a statistical model that describes the data of interest. We then formulate a prior distribution over the unknown parameters of the model, which is meant to express our beliefs about the situation before having the data. After observing some data, we then apply Bayes' Theorem to obtain a posterior distribution for these unknowns, which takes account of both the prior and the data [21]. From this posterior distribution we can compute predictive distributions for future observations. What is the Bayes' Theorem?

3.1.1 Bayes' Theorem

Suppose that $y' = (y_1, \ldots, y_N)$ is a vector of N observations whose probability distribution $p(y|\theta)$ depends on the values of k parameters $\theta' = (\theta_1, \ldots, \theta_k)$. Suppose also that θ itself is a random quantity having a probability distribution $p(\theta)$. Then using the probability multiplication theorem, we have,

$$p(y \mid \theta) p(\theta) = p(y \text{ and } \theta) = p(\theta \mid y) p(y) \tag{3.1}$$

Once the observed data y are available, the conditional distribution of θ given y can be written in form (3.2) as,

$$p(\theta \mid y) = \frac{p(y \mid \theta) p(\theta)}{p(y)} \tag{3.2}$$

where

$$p(y) = c^{-1} = \int p(y \mid \theta) p(\theta) d\theta \qquad \theta \text{ is continuous}$$
$$= \sum p(y \mid \theta) p(\theta) \qquad \theta \text{ is discrete}$$

Here the sum or the integral is taken over the admissible range of θ and c is called a "normalizing" constant necessary to ensure that $p(\theta|y)$ integrates or sums to 1. Thus we may write (3.2) alternatively as

$$p(\theta \mid y) = cp(y \mid \theta) p(\theta) = \frac{p(y \mid \theta) p(\theta)}{\int p(y \mid \theta) p(\theta) d\theta} \tag{3.3}$$

We refer to form (3.2) or its equivalent (3.3) as Bayes' Theorem. In this expression, $p(\theta)$ is called the prior distribution of θ and it tells us what is known about the unknown parameter θ before the data are available. Correspondingly, $p(\theta|y)$, is called the posterior distribution of θ given y and it tells us what is known about θ after the data are available. Often, the prior distribution of θ

depends on some other vector of hyperparameters η and we could express this hyperparameter in the prior distribution written as $\pi(\theta \mid \eta)$. Using this notation, the inference about θ is then based on the posterior distribution of the form (3.4),

$$p(\theta \mid y, \eta) = \frac{p(y \mid \theta)\pi(\eta)}{\int p(y \mid \theta)\pi(\theta \mid \eta)d\theta} \tag{3.4}$$

3.1.2 Likelihood Function

Suppose that we have a sample of N observations y with which we associate an N-dimensional random variable $y_1, y_2, \ldots, y_N$, whose probability distribution $p(y \mid \theta)$ is known and depends on some unknown parameter θ. Before the data are available, $p(y \mid \theta)$ will associate a density with each different outcome of y of the experiment, for fixed θ. After the data or the sample values have been obtained, we are led to think about the various values of θ that might have produced the fixed set of observations y we actually have in hand. The appropriate function for this purpose is called the likelihood function and can be written as $L(\theta \mid y)$. The likelihood function is of the same form as the probability distribution except that the data is now thought of as fixed but the parameter is variable.

For example, suppose **Y** is a random variable distributed as a normal distribution with mean μ and the standard deviation σ. If **y**= $(y_1, y_2, \ldots, y_N)$ is a random sample of **Y**, then the likelihood function of the sample is

$$L(\mu, \sigma \mid y_1, y_2, \ldots, y_N) = \left(2\pi\sigma^2\right)^{-N/2} \exp\left(-\frac{1}{2}\sum_{i=1}^{N}\left(\frac{y_i - \mu}{\sigma}\right)^2\right) \tag{3.15}$$

3.1.3 Bayes' Theorem and the Likelihood Function

The likelihood function $L(\theta \mid y)$ plays a very important role in Bayes' theorem because after the data has come in hand, $p(y \mid \theta)$ in the right hand side of (3.2), which may be regarded as a function not of y but of θ, is the likelihood function we mentioned earlier. In other words, Bayes' theorem says that, the posterior probability distribution for θ is proportional to the product of the prior distribution of θ and the likelihood function for θ given y. That is,

$$\text{posterior distribution} \propto \text{likelihood} \times \text{prior}$$

We can see now that both the experimental data and the prior opinion will make contributions to the posterior distribution of the unknown parameter θ. Data will modify the prior knowledge θ through the likelihood function, which can thus be regarded as representing the information about θ coming from the data.

The Bayesian approach to statistical inference is quite different from that of the classical, or frequentist statistical philosophy. The frequentist evaluates procedures based on repeated sampling, imagining an infinite replication of the same inferential problem and evaluating properties over this repeated sampling framework for fixed values of unknown parameters. While in the Bayesian approach, with a sampling model or the likelihood function and a prior distribution, unknown parameters are also considered as random and all inferences are based on their distribution conditional on the observed data (the posterior distribution). The Bayesian evaluates procedures for an infinite sampling experiment of parameters drawn from the posterior distribution for a given data set. Simply put, the frequentist conditions on parameters and replicates over the data, while the Bayesian conditions on the data and replicates (integrates) over the parameters [22].

3.2 Bayesian Decision Analysis

In Bayesian decision analysis, suppose that we have a set of available actions $A = (a_1, a_2, \ldots, a_r)$ from which a choice has to be made. For each action, there is a loss (or payoff) which depends on a state of nature, say the unknown set of parameters θ whose possible outcomes are denoted by Θ. And it is easy to imagine quantifying this loss (or payoff) with a function, say a loss function $l(\theta,a)$, which gives the loss incurred when θ is the true state of nature and we take action a. The Bayesian decision analysis of choosing an action is to use a posterior distribution of θ which combines prior knowledge of θ with the information provided by the data, as given in (3.3). Given the posterior distribution, we then choose the action which minimizes the expected loss over the posterior distribution. Therefore in order to set up the general Bayesian decision analysis, we need the following components: A prior distribution, a sampling distribution (the likelihood), a class of allowable actions and a loss function.

3.2.1 Prior Distribution

Implementation of the Bayesian approach depends on a willingness to assign probability distributions not only to data variables, but also to parameters like θ. The latter is called a prior distribution. There are several ways to specify a prior distribution. We can estimate priors from a manageable collection of possible θ values based on information accumulated from the past studies. We can choose a conjugate prior which will simplify the future analysis by making the posterior distribution $p(\theta \mid y)$ belonging to the same distributional family specified for the prior. We can also choose a noninformative prior. In the last case, the inference will be based solely on the data collected. Bayesian analysis with noninformative priors has been remarkably successfully in some cases. Another method of constructing priors, which we shall use in this project, is the method called empirical estimation of the prior, and this is the method used in empirical Bayes (EB) analysis.

3.3 Factor Group Assignment via Bayesian Decision Theory

Suppose in a small jurisdiction with a small set of permanent highway traffic classification count sites, each of the WIM sites has been identified as a separate factor group. Each factor group is characterized by certain properties, say the monthly and day-of-week variation pattern. A factor group assignment problem consists of collecting short-count classification traffic samples from a road whose state of nature is unknown, and assigning this short-count site, by some sort of assignment criteria, to one factor group, whose characteristic can then be used in lieu of the unknown state of nature to estimate the mean daily traffic by vehicle type for the short-count site. In fact, even before any short-count data are available, it is possible to have an opinion regarding the plausibility of assigning a given site to each of a set of factor groups based on the geographical location of the site, the functional class of the road, or some other previous experience. It is also reasonable to quantify this assignment opinion by giving each factor group in the set of a probability value, with a high probability being given to a factor group to which one believes that it is more likely that the short-count site belongs. If the situation happens to be that one is totally uncertain as to the membership assignment, an equal probability could be assigned to each factor group. Classification traffic counts, when available, provide additional information concerning the factor group membership and one could then judge his or her prior opinions on the factor group membership, and modify it accordingly. This process of modifying prior viewpoints in the light of data in the most efficient manner is the subject of Bayesian statistics. A decision problem could be set up for the factor group assignment, and Bayes' theorem can be used to compute the optimal updating of the prior probability assessment after obtaining new data.

3.3.1 Factor Group Assignment

To provide a formal statement of the factor group assignment problem, imagine that we have in hand a total of n different factor groups showing distinct patterns of monthly and day-of-week variations in daily classification traffic counts. That is, each of the three vehicle classes for each of these groups is characterized by a set of monthly and day-of-week multipliers $M_i^{(k)}$ and $W_j^{(k)}$. Here i =1,...,12, j=1,...7, and k=1,..,3. Then we define a column vector β_i, one for each WIM factor group, by stacking the natural logarithms of the multipliers, $m_i^{(k)}$ and $w_j^{(k)}$ as defined in equation (2.3). The autoregressive coefficient matrix obtained from the multiplicative time series model defined in equation (2.5) are also set in Φ_i, one for each factor group. Also available from the model fitting stage is the noise covariance matrix Σ_i, one for each factor group. Now suppose we have also obtained a sequence of N classification daily traffic counts $\mathbf{z}=[\mathbf{z}_1,\mathbf{z}_2, \ldots, \mathbf{z}_i, \ldots, \mathbf{z}_N]$ collected at some highway site. Although for convenience these daily counts are indexed by

1,2,...,N, there is no assumption made that the counts actually come from consecutive days, and could in fact be generated by an arbitrary sampling plan. Now let β, μ, Φ and Σ denote the vector of monthly and day-of-week adjustment terms, mean daily traffic by vehicle type, the autoregessive coefficient matrix and the covariance matrix, appropriate for the highway site where the short count was taken. By assumption β must be matched to one of the known vectors β_i to decide the factor group membership. Since our ultimate goal in obtaining the factor group assignment is to use this method to develop a more accurate estimator of MDT by vehicle classifications, the parameter of interest is μ while the rest, Φ and Σ are the so-called "nuisance parameters". The estimates of Φ_i and Σ_i for each of the 24 yearly data sets from the daily classification traffic count modeling in Chapter 2 should provide information concerning the likely values for the nuisance parameters under the assumption that those WIM sites are representative of other highway sites. And it is desirable to exploit this information to sharpen the decision concerning the factor group membership and simplify the analysis later on. Therefore, we deploy an empirical Bayes analysis by using estimates of Φ_i and Σ_i. Denoting θ_i to be an array containing the adjustment terms β_i, the autoregressive terms Φ_i and the noise covariance matrix Σ_i, i.e., $\theta_i=[\beta_i, \Phi_i, \Sigma_i]$, one for each WIM factor group, and $\theta=[\beta, \Phi, \Sigma]$ to be those terms appropriate for the highway site, our inference is then to assign θ to one of the existing θ_i.

3.3.2 Prior Classification Probabilities

We can establish classification probabilities α_i, $i=1,\ldots,n$, one for each factor group, prior to the classification traffic data, such that α_i is the our opinion of the probability that θ equals θ_i, i.e.,

$$\alpha_i = \text{Prob}[\,\theta=\theta_i\,]$$

In most practical applications, the prior distribution of classification probabilities can be taken to be uniformly distributed as $\alpha_i = 1/n$, $i=1,\ldots,n$, indicating complete prior uncertainty as to which group the site belongs. If one has strong beliefs as to the site's group membership, it can be captured in a non-uniform α_i.

3.3.3 Loss Function

A "0-1" loss function, denoted by $l(\theta,\theta_i)$, which gives the penalty incurred if we select θ_i as the terms for the highway short-count site can be given as,

$$l(\theta,\theta_i) = \begin{cases} 0, & \text{if we select } \theta_i \text{ and } \theta \text{ actually equals } \theta_i, \\ 1, & \text{if we select } \theta_i \text{ but } \theta \text{ actually equals some other } \theta_l,\ l \neq i. \end{cases}$$

The interpretation of the above form is, if the site actually belongs to group i, and we assign it to group i, we will not incur any loss, while if the site belongs to some other group l and we assign it to i, we incur a loss of 1.0.

After collecting daily classification traffic data $\mathbf{z}=[\mathbf{z}_1,\mathbf{z}_2, \ldots, \mathbf{z}_i, \ldots, \mathbf{z}_N]$, i=1,...,N, and denoting $\mathbf{y}=(\mathbf{y}_1,\mathbf{y}_2,\ldots, \mathbf{y}_i, \ldots, \mathbf{y}_N)'$ as the natural logarithm of the traffic counts $\mathbf{z}$ with each $\mathbf{y}_i$ is a 3×1 column vector containing the classification daily counts as, $\mathbf{y}_i = (y_i^{(1)}, y_i^{(2)}, y_i^{(3)})'$, we can formulate the Bayesian posterior distribution of the parameter θ,

$$\text{prob}[\ \theta = \theta_i \mid \mathbf{y}\ ] = \frac{f(\mathbf{y} \mid \theta = \theta_i)\alpha_i}{\sum_{l=1}^{n} f(\mathbf{y} \mid \theta = \theta_l)\alpha_l} \tag{3.6}$$

The expected posterior loss for selecting θ_i as the adjustment vector can then be shown to be,

$$E[\ l(\theta,\theta_i)] = \sum_{l=1}^{n} l(\theta,\theta_l)\text{prob}[\ \theta = \theta_l \mid \mathbf{y}\ ] = 1 - \text{prob}[\ \theta = \theta_i \mid \mathbf{y}\ ] \tag{3.7}$$

For each vector θ_i, the right-hand side of the above equation gives the loss we can expect to incur if we use θ_i after observing the data sample y. Notice that it is actually one minus the posterior classification probability. Since our objective is to minimize the expected posterior loss, then the optimal decision rule is to set θ equal to the θ_i having the largest posterior probability. Thus using "0-1" loss function, leads to a particularly tractable and intuitively meaningful result, i.e, the adjustment vector θ_i having the largest posterior probability is the optimal estimate of the unknown state of nature θ for the short-count site.

3.3.4 Likelihood Function

The Bayesian approach requires a sampling distribution, or the likelihood function to evaluate the probability density $f(y|\theta)$ shown in posterior probability distribution form the (3.6). So a set of likelihood functions, one for each factor group, denoted by $f(y_1,y_2,\ldots,y_N|\theta=\theta_i)$, which gives the probability (or the probability density) of obtaining the logarithm of the count sample had the site actually belonged to group i needs to be specified. The model development work described in Chapter 2 provides a starting point to specify the likelihood functions. If we restrict our attention to one-direction classification traffic volumes on the state of Minnesota highways, and if we assume that the WIM sites are representative of all relevant non-WIM sites, then the results from Chapter 2 imply that the daily classification traffic counts at the site of interest can be regarded as being generated by a stochastic process described by a multivariate lognormal regression model in equation (2.4), with error terms following a multivariate autoregressive model of the form described in equation (2.5). Let $\mu=(\mu^{(1)},\mu^{(2)},\mu^{(3)})'$ be a column vector containing the logarithm of the mean daily classification traffic for the short-count site and **V** be the covariance matrix for the count sample **y**, which can be computed once one knows the value of the multivariate AR(1) coefficients Φ, the noise covariance matrix Σ and the sampling plan. The likelihood function of the sample can now be computed using the appropriate multivariate normal density in (3.8)

$$f(\mathbf{y}\,|\,\theta=\theta_i,\mu)=(2\pi)^{-3N/2}|V_i|^{-1/2}\exp\left(-\frac{1}{2}(\mathbf{y}-\mathbf{X}\beta_i-\mathbf{1}_{3N\times3}\mu)'V_i^{-1}(\mathbf{y}-\mathbf{X}\beta_i-\mathbf{1}_{3N\times3}\mu)\right) \quad (3.8)$$

Matrix **X** and $\mathbf{1}_{3N\times3}$ are defined as in equation (2.4). To clarify the notation, we emphasize that **X** is a matrix of dimension $3N \times (3\times19)$ and $\mathbf{1}_{3N\times3}$ is a matrix of dimension $3N\times3$, as follows,

$$\mathbf{X} = \begin{bmatrix} \tilde{\mathbf{X}}_1 \\ \tilde{\mathbf{X}}_2 \\ \vdots \\ \tilde{\mathbf{X}}_N \end{bmatrix} \qquad \text{with} \quad \tilde{\mathbf{X}}_t = \begin{bmatrix} \mathbf{X}_t & \vec{\mathbf{0}}^T & \vec{\mathbf{0}}^T \\ \vec{\mathbf{0}}^T & \mathbf{X}_t & \vec{\mathbf{0}}^T \\ \vec{\mathbf{0}}^T & \vec{\mathbf{0}}^T & \mathbf{X}_t \end{bmatrix}$$

and

$$\mathbf{1}_{3N\times 3} = \begin{bmatrix} \mathbf{I}_3 \\ \mathbf{I}_3 \\ \vdots \\ \mathbf{I}_3 \end{bmatrix} \qquad \text{with} \quad \mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $\mathbf{X}_t$ is a row vector of size 19 and whose elements contain only 0 or 1 depending on the month and day-of-week of the daily count on day t.

3.4 Posterior Classification Probability

At this point, the likelihood functions in equation (3.6) depend only on knowing factor group membership, while the likelihood in equation (3.8) requires this plus the knowledge of the site-specific parameter μ. The Bayesian method of deriving the likelihoods appearing in form (3.6) from the likelihood in (3.8) is to multiply (3.8) by the prior probability distribution for μ to produce the joint distribution of the parameters and sample data, and then integrating out the site-specific parameter μ from this joint distribution to obtain the "marginal" likelihoods appearing in equation (3.6). The prior distribution for μ needs also to be specified. In this project, we take the prior of μ to be locally uniform and thus employ the commonly used "noninformative" prior for μ, as illustrated in Box and Tiao [23]. That is,

$$\pi(\mu) \propto c, \; c = \text{constant}$$

And therefore the likelihood function in (3.6) can be rewritten as in equation (3.9)

$$f(\mathbf{y} \mid \theta = \theta_i) = \int_{R} f(\mathbf{y} \mid \theta = \theta_i, \mu)\pi(\mu)d\mu \tag{3.9}$$

where R denote the appropriate region of μ.

This noninformative prior for the location parameter is reasonable because this will ultimately let the posterior estimate of MDT, which will be explained in detail in Chapter 4, to be determined primarily by the observed data.

Let

$$\tilde{\mathbf{y}}_i = \mathbf{y} - \mathbf{X}\beta_i \tag{3.10}$$

and

$$SS_i = (\tilde{\mathbf{y}}_i - \mathbf{1}_{3N\times 3}\mu)'V_i^{-1}(\tilde{\mathbf{y}}_i - \mathbf{1}_{3N\times 3}\mu) \tag{3.11}$$

Then we could find a Weighted Least Squares estimator of μ as,

$$\hat{\mu}_i = \left(\mathbf{1}'_{3N\times 3}V_i^{-1}\mathbf{1}_{3N\times 3}\right)\left(\mathbf{1}'_{3N\times 3}V_i^{-1}\tilde{\mathbf{y}}_i\right) \tag{3.12}$$

By adding and subtracting the same term $\mathbf{1}\hat{\mu}$ from the $(\tilde{\mathbf{y}}_i - \mathbf{1}\mu)$ and $(\tilde{\mathbf{y}}_i - \mathbf{1}\mu)'$ in (3.11), we get an alternative form of the likelihood function with the μ term being separated from others,

$$f(\mathbf{y} \mid \theta = \theta_i, \mu) = (2\pi)^{-3N/2}|V_i|^{-1/2}\exp\left(-\frac{1}{2}(\tilde{\mathbf{y}}_i - \mathbf{1}_{3N\times 3}\hat{\mu})V_i^{-1}(\tilde{\mathbf{y}}_i - \mathbf{1}_{3N\times 3}\hat{\mu})\right)\exp\left(-\frac{1}{2}(\mu - \hat{\mu})(\mathbf{1}'_{3N\times 3}V_i^{-1}\mathbf{1}_{3N\times 3})(\mu - \hat{\mu})\right) \tag{3.13}$$

Multiplying the right hand side of (3.13) by the prior distribution of μ and applying the useful integral formulae in Box and Tiao [23], we obtain, after canceling out terms that are common to the numerator and denominator, the formula for the posterior classification probabilities for multiple count categories,

$$\text{Prob}[\ \theta = \theta_i \mid \mathbf{y}\,] = \frac{s_i S_i \alpha_i}{\sum_l^n s_l S_l \alpha_l} \tag{3.14}$$

where

$$\begin{aligned} s_i &= |V_i|^{-1/2} \left| \mathbf{1}'_{3N\times 3} V_i^{-1} \mathbf{1}_{3N\times 3} \right|^{-1/2} \\ S_i &= \exp\left(-\frac{1}{2} (\tilde{\mathbf{y}}_i - \mathbf{1}_{3N\times 3} \hat{\mu}_i)' V_i^{-1} (\tilde{\mathbf{y}}_i - \mathbf{1}_{3N\times 3} \hat{\mu}_i) \right) \end{aligned} \tag{3.15}$$

As noted earlier, the covariance matrix **V** is a function of the autoregressive coefficient matrix Φ, the noise covariance matrix Σ and the sampling plan. It is of dimension $3N \times 3N$. To determine the submatrices of **V**, that is, $\Gamma(h)$, the 3×3 covariance matrix for lag h revealed by the traffic count sample, one could recursively use the Yule-Walker equations in (3.16) if the autoregressive coefficient matrix Φ and the covariance matrix $\Gamma(0)$ of the series are known [24].

$$\Gamma(0) = \Phi \Gamma(0) \Phi' + \Sigma \tag{3.16}$$

$\Gamma(0)$ can be computed using the Lyaponov equation in (3.17) or its equivalent form in (3.18), once we obtain the coefficient matrix Φ and the noise covariance matrix Σ from the model fitting stage described in Chapter 2.

$$\Gamma(h) = \Phi \Gamma(h-1) \tag{3.17}$$

or

$$vec\ \Gamma(0) = vec\ (\Phi\Gamma(0)\Phi') + vec\ \Sigma = (\Phi \otimes \Phi) vec\ \Gamma(0) + vec\ \Sigma \qquad (3.18)$$

3.5 Evaluation of the Classification Methodology for Factor Group Assignment

The evaluation of the methodology we introduced earlier requires as input the monthly and day-of-week adjustment terms β, the autoregressive coefficient matrix Φ and the noise covariance matrix Σ for each factor group, and in Chapter 2 we described how estimates of these quantities were obtained for WIM site factor groups for 1992, 1993, 1994 and 1995. Table 3.1 lists the site identification number for the factor groups in years 1992 to 1995.

To perform the empirical evaluation of the assignment algorithm, we first pick the sites which we have data available through several years, for example, site 1023 and 4033 in 1994 and 9075 in 1995, as the test sites, and then generated a random 12-day count sample, one count from each month for these sites. With the help of *EBCLASM*, a program written in S-Plus, which is attached in Appendix B, we computed the posterior classification probabilities for these sites using the estimates of monthly and day-of-week adjustment factors in other years. Table 3.2-a and 3.2-b displays the posterior classification probabilities for site 1023 (94), 4033 (94) and 9075 (95), to factor groups in year 1993 and 1992, respectively. Assuming these factor groups for each year are well-constructed, and that the underlying traffic generation patterns were similar through these years, then the latter year informative samples should give the greatest posterior probability to the factor group having the same WIM site identification number in 1992, 1993. Inspection of Table 3.2-a and 3.2-b shows that this was true for most cases. Site 1023 (94) and 4033 (94) were assigned with the highest probability (almost 1.0) to their matching sites in year 1992 and 1993. And site 9075 (95) was also assigned to the factor group 9075 in year 1994 with a probability of 1.0. However, one exception occured for site 9075 (95) when assigned to factor groups in 1992. Notice that in this case, the monthly and day-of-week adjustment terms we used were about three years in the past. A one day sample from each month thus appears to reliably classify a site as long as the adjustment terms are relatively recent.

Table 3.1 Factor Group Assignment in 1992, 1993, 1994 and 1995

Factor Group Identification Numbers			
1992	**1993**	**1994**	**1995**
1019		1019	1019
1023	1023	1023	
1029		1029	
1085			1085
4033	4033	4033	
	4037	4037	
4040	4040		
		4055	
6251	6251	6251	
9075	9075		9075

Table 3.2-a Evaluation of Classification Methodology using Factor Groups in 1993

Classification Probabilities			
93 Factor Group Site ID	Test Site 1023 (94)	Test Site 4033 (94)	Test Site 9075 (95)
1023	9.999e-001	2.647e-005	3.669e-012
4033	1.998e-026	9.997e-001	0.000e000
4037	2.550e-020	1.591e-005	4.233e-145
4040	4.224e-008	2.635e-008	2.835e-013
6251	6.650e-005	6.146e-006	8.060e-011
9075	1.951e-007	2.179e-004	1.000e000

Table 3.2-b Evaluation of Classification Methodology using Factor Groups in 1992

Classification Probabilities			
92 Factor Group Site ID	Test Site 1023 (94)	Test Site 4033 (94)	Test Site 9075 (95)
1019	3.369e-020	1.142e-011	1.000e000
1023	9.999e-001	1.399e-002	1.0357e-031
1029	1.327e-015	3.616e-004	2.227e-022
1085	1.044-e012	2.308e-005	1.952e-021
4033	1.837e-043	9.853e-001	0.000e000
4040	1.404e-009	3.162e-004	9.681e-025
6251	1.603e-007	8.573e-006	3.427e-025
9075	7.611e-015	4.314e-005	3.376e-019

We also computed the factor group assignment probabilities using the classification methodology for 2-day samples. Tables 3.3 and 3.4 listed the average classification probabilities resulted from 2-day count samples in 1994 using the adjustment terms in 1992 and 1993, respectively. The classification probabilities listed here are averaged over all the 2-day samples available.

Table 3.3 Classification Probabilities to 1992 Factor Group using 1994 2-day Samples

Classification Probabilities							
92 Factor Group Site ID	1019 (94)	1023 (94)	1029 (94)	4033 (94)	4037 (94)	4055 (94)	6251 (94)
1019	0.192	0.143	0.119	0.091	0.092	0.180	0.132
1023	0.063	0.069	0.078	0.062	0.065	0.063	0.070
1029	0.105	0.101	0.074	0.081	0.092	0.087	0.086
1085	0.355	0.284	0.389	0.312	0.302	0.280	0.312
4033	0.016	0.029	0.011	0.050	0.046	0.024	0.024
4040	0.089	0.087	0.072	0.087	0.077	0.074	0.083
6251	0.070	0.082	0.073	0.109	0.101	0.084	0.084
9075	0.111	0.205	0.185	0.208	0.225	0.209	0.210

Table 3.4 Classification Probabilities to 1993 Factor Group using 1994 2-day Samples

Classification Probabilities							
93 Factor Group Site ID	1019 (94)	1023 (94)	1029 (94)	4033 (94)	4037 (94)	4055 (94)	6251 (94)
1023	0.078	0.311	0.067	0.083	0.083	0.132	0.248
4033	0.054	0.025	0.156	0.121	0.118	0.067	0.015
4037	0.073	0.048	0.109	0.100	0.108	0.092	0.083
4040	0.360	0.103	0.326	0.266	0.298	0.216	0.246
6251	0.107	0.333	0.103	0.068	0.083	0.078	0.352
9075	0.328	0.186	0.249	0.362	0.310	0.415	0.056

3.6 Conclusion

In this chapter, we first introduced the Bayesian Decision Theroy and then set up the factor group classification problem using Bayesian Decision Theory. From the empirical evaluation of this methodology, we can see from the results shown in Table 3.2 that this algorithm worked well by assigning the sites to the factor groups having the same site number in another year using a 12-day (one day frrom each month) sample. When using a two-day sample however, accurate assignment is very difficult, as can be seen from Table 3.3 and Table 3.4. This motivates us to think about estimating classified Mean Daily Traffic as weighted average across all the factor group sites, which we shall discuss at length in Chapter 4.

CHAPTER 4

BAYES ESTIMATION OF CLASSIFIED MEAN DAILY TRAFFIC

In chapter 3, we described a method for assigning short count locations to WIM factor groups based on the information contained in classification traffic count samples, where the classification rule was derived using principles of Bayesian decision theory. In addition, we evaluated our method using the LTPPP Minnesota WIM dataset and obtained the classification probabilities of the short count site to the WIM factor groups. Those results are satisfactory. We also pointed out in Chapter 3 that accurate factor group assignment was not an end in itself, but a means for developing an accurate estimates of MDT by vehicle class. The most obvious way to use the Bayesian methods of Chapter 3 is to first compute the posterior classification probabilities using (3.15--3.16), and then use the monthly and day-of-week factors characterizing the group with the highest posterior classification probability to adjust the short count sample. It may be however that the appropriate factor group membership for a site may remain uncertain even after obtaining a count sample, so that one can not say with high confidence which set of adjustment factors are appropriate. Also, because this method does not take into account uncertainty arising from mis-classification, the precision of the resulting classified MDT estimates are likely to be overstated. Finally, there is no reason to believe that the resulting classified MDT estimates are the best that can be obtained from a given traffic classification sample. In this chapter we will develop a method for computing Bayes estimates of MDT of multiple count categories, which allows for posterior uncertainty concerning factor group membership and is optimal in the sense that on average they will be closer to the true classified MDT than other estimators.

4.1 Bayes Estimation of Classified MDT

Developing a Bayes estimator of classified Mean Daily Traffic (MDT) proceeds in a manner similar to that described in Chapter 3. That is, we need some prior ideas concerning the likely values of the classified MDT, a likelihood function and a loss function. Then by combining the prior view with the data likelihood, we could obtain the posterior probability distribution which characterizes the uncertainty about a site's MDT after the classification traffic count sample becomes available.

4.1.1 Likelihood Function

As was stated in Chapter 2, the starting point of our analysis is that the expected daily traffic volume of day t for multiple count categories, $E[\mathbf{z}_t]$, occurring during month i and day-of-week j, is given by,

$$E\left[\mathbf{z}_t\right] = \mathbf{z}_0 M_i W_j \tag{4.1}$$

where M_i is the monthly multiplier for month i, W_j is the day-of-week multiplier for day j, for the appropriate count category, and $\mathbf{z}_0$ is the classified mean daily traffic. In Chapter 2 it was shown that multivariate lognormal regression models in (2.3) provide defensible statistical models for the classification daily traffic volumes. Based on this, we can write down,

$$\mathbf{z}_t = \exp(\mathbf{y}_t) \sim \Lambda(\mu + \tilde{\mathbf{X}}_t \beta, \Gamma_0) \tag{4.2}$$

where "~" reads as "distributed", and the notation Λ represents the lognormal distribution. Then the above expression tells us that the random variable $\mathbf{z}_t$ is distributed as a lognormal distribution with mean $\mu + \tilde{\mathbf{X}}_t \beta$, and a covariance matrix Γ_0, as were defined in Chapter 3. The lognormal distribution model implies that if a classification daily count $\mathbf{z}_t$ was made during month i and on day-of-week j, the expected classification traffic count $E[\mathbf{z}_t]$ can be given by,

$$E\left[\mathbf{z}_t\right] = \left(e^{\mu + \gamma/2}\right)\left(e^{m_i}\right)\left(e^{w_j}\right) = \exp\left(\mu + \frac{\gamma}{2} + \tilde{\mathbf{X}}_t \beta\right) \tag{4.3}$$

where γ is a column vector containing the diagonal elements of the covariance matrix Γ_0, that is, $\gamma = (\Gamma_{0,11}, \Gamma_{0,22}, \Gamma_{0,33})'$. And $z_0 = \exp(\mu + \gamma/2)$ is the expected classification daily traffic volume on a typical day, i.e., the classified MDT.

4.1.2 Prior Distribution

Since z_0 is a function of μ and Γ_0 which is in turn a function of the autoregressive coefficient matrix Φ and the noise covariance matrix Σ, assuming a prior distribution for these parameters in turn induces a prior distribution for z_0. In Chapter 3, we described how a noninformative prior for μ could be combined with empirical priors for Φ and Σ to calculate the posterior classification probabilities. And in the Bayes estimation of classified MDT, we will still adopt these priors for μ, Φ and Σ. That is, we shall use a noninformative prior for μ and empirical informative priors developed from the results of the model fitting for the site autocorrelation coefficient matrix Φ and noise covariance matrix Σ.

4.1.3 Loss Function

In Bayesian analysis, the Bayes rule states that Bayes decision rule is chosen to minimize the posterior risk [22]. Under the squared error loss between the estimate and the true MDT, and by applying Bayes rule, it turns out to be that the point estimate of classified MDT which minimizes the expected loss is simply the mean of the posterior distribution. And this would be our Bayes estimator of classified MDT. Thus we will use the squared error loss function as the loss function, which can be expressed as,

$$l(z_0, \hat{z}) = (z_0 - \hat{z})^2 \tag{4.4}$$

where $\hat{z}$ is an estimate of the true classified MDT z_0, and we will seek the estimator which minimizes the posterior expected squared error loss, i.e,

$$E[l \mid \mathbf{y}] = E[(z_0 - \hat{z})^2 \mid \mathbf{y}] \tag{4.5}$$

Thus our Bayes estimator of classified MDT, which we shall denote by $\mathbf{z}_B$, can be interpreted as producing estimates which can be expected to be closer to the true classified MDT than any other estimates, given the available data. Since it is well known that the estimator $\mathbf{z}_B$ minimizing (4.5) is the posterior expected value of $\mathbf{z}_0$ [25], which can be expressed as follows,

$$z_B = E[z_0 \mid \mathbf{y}] = \int_R z_0 f(z_0 \mid \mathbf{y}) dz_0 \tag{4.6}$$

this estimation problem is in principle solved one if we could determine the posterior distribution of $\mathbf{z}_0$ given the data, that is $f(\mathbf{z}_0 \mid \mathbf{y})$.

4.1.4 Posterior Distribution of Classified MDT

Rather than attempt to characterize the posterior probability density of $\mathbf{z}_0$ and then compute the $\mathbf{z}_B$ via equation (4.6), the discrete nature of the empirical Bayes prior allows a more direct derivation. To begin with, suppose that we know the site of interest belongs to factor group i, and that array $\theta_i = [\ \beta_i, \Phi_i, \Sigma_i\ ]$ contains the correct values for the parameters of group i. According to Bayes theorem,

$$f(\mu \mid \mathbf{y}, \theta = \theta_i) = \frac{f(\mathbf{y} \mid \mu, \theta_i) f(\mu \mid \theta_i)}{\int_R f(\mathbf{y} \mid \mu, \theta_i) f(\mu \mid \theta_i) d\mu} \tag{4.7}$$

Using the appropriate multivariate normal density for $f(\mathbf{y} \mid \boldsymbol{\mu}, \theta_i)$ and noninformative prior distribution for $\boldsymbol{\mu}$, and then factoring the normal distribution and applying some useful integral techniques, we finally obtain an alternative form of (4.7) as,

$$f(\mu \mid \mathbf{y}, \theta_i) = (2\pi)^{-3/2} \left|\tilde{V}_i\right|^{-1/2} \exp\left(-\frac{1}{2}(\mu - \hat{\mu}_i)' \tilde{V}_i^{-1} (\mu - \hat{\mu}_i)\right) \tag{4.8}$$

where

$$\tilde{V}_i^{-1} = \mathbf{1}_{3N\times 3}{}' V_i^{-1} \mathbf{1}_{3N\times 3} \tag{4.9}$$

The definitions appeared in (3.11) and (3.13) also apply to (4.8). From there we can see that $f(\mu \mid \mathbf{y}, \theta_i)$ is distributed as normal distribution with mean $\hat{\mu}_i$ and covariance matrix $\tilde{V}_i$, that is $f(\mu \mid \mathbf{y}, \theta_i) \sim N(\hat{\mu}_i, \tilde{V}_i)$. Therefore,

$$f\left(\left(\mu + \frac{1}{2}\gamma_i\right) \mid \mathbf{y}, \theta_i\right) \sim N\left(\hat{\mu}_i + \frac{1}{2}\gamma_i, \tilde{V}_i\right) \tag{4.10}$$

And this implies that the classified mean daily traffic $\mathbf{z}_0 = \exp(\mu + \gamma/2)$ given the classification count data $\mathbf{y}$ and that the site belongs to factor group i, is distributed as a lognormal distribution with mean $\hat{\mu}_i + \frac{1}{2}\gamma_i$ and covariance matrix $\tilde{V}_i$. Using the empirical estimates of the priors for Φ and Σ, the expected classified MDT given the sample and that the site membership to group i can be expressed as,

$$\bar{\mathbf{z}}_i = E[\mathbf{z}_0 \mid \mathbf{y}, \theta_i] = \exp\left(\hat{\mu}_i + \frac{1}{2}\gamma_i + \frac{1}{2}\nu_i\right) \tag{4.11}$$

where ν is a column vector containing the diagonal elements of the covariance matrix $\tilde{V}_i$. Finally, the expected classified MDT given only the sample can be computed using,

$$\mathbf{z}_B = E[\mathbf{z}_0 \mid \mathbf{y}] = \sum_{i=1}^{n} \bar{z}_i \text{Prob}\,[\theta = \theta_i \mid \mathbf{y}] \tag{4.12}$$

The intermediate quantities appearing in (4.12) are identical to those used in computing posterior classification probabilities, as described in section 3.4, so that computing a Bayes estimates of classified MDT can follow the same procedure as computing the classification probabilities.

4.2 Evaluation of Bayes Estimator of Classified MDT

The performance of applying Bayesian decision theory to obtain Bayes estimates of classified MDT was evaluated using the WIM dataset in the following way. The LTPPP Minnesota's 1994 WIM data from seven sites were used as the test data, and these same seven sites also composed the factor group in the current year. Six WIM sites in the year of 1993 composed the factor group in the preceding year and eight WIM sites in the year of 1992 composed the historic factor group. Please refer to Table 3.1 for the factor group identification number for each year. In this way, for each sample we draw from the 1994 WIM site, we can calculate three Bayes estimates of classified MDT, the first is the MDT based on the current year (year 1994) adjustment factors, the second is based on the preceding year (year 1993) adjustment factors and the last one is based on the 2-year old historic (year 1992) adjustment factors. We shall call them "current year Bayes" estimator, "preceding year Bayes" estimator and "historic year Bayes" estimator. When calculating the current year Bayes estimates of classified MDT for a certain WIM site, that site was left out of the pool of the factor group. All these three Bayes estimators of classified MDT were compared to two other estimators, the first one was a "Factored Average" of the counts in a sample, in which the true monthly and day-of-week adjustment factors for this site computed in Chapter 2 were assumed known and actually used to adjust the classification counts. That is, each sample of the traffic classification counts was assigned to the factor group from which counts were actually drawn, and then each classification count was divided by the appropriate monthly and day-of-week multipliers characteristic of that group. These factored counts for each category were then averaged to estimate the classified MDT. Because of this scenario, we shall call this estimator as the "perfect" estimator in what follows. The other estimator was the simple average of the classification counts in a sample, without any factoring or any other adjustment. We shall call it "unadjusted" estimator. Three sampling strategies are used in the evaluation of the performance of the estimators, they are, 2-day samples, one-full week samples and two full-week samples. As a reference, the AADT by classification for each of the test sites was computed from all good data from that year, using the *AASHTO* [1] successive averaging method, which accounts for much of the bias caused by missing data. Absolute percent errors, the absolute value of PE as defined in absolute value of PE as defined in equation (4.13),

$$APE = |PE| = \left| \frac{\hat{z} - AADT}{AADT} \times 100 \right| \tag{4.13}$$

were then computed for each estimator and each sampling plan as an index to evaluate the performance of each estimator. Then the performance of each estimator for three different sample sizes was summarized in the plot of the accumulated proportions of the absolute percent errors.

Table 4.1 Mean Estimation Error and Standard Deviation for Different Estimators

Mean Estimation Error and Standard Deviation							
Estimator	**Sample Size**	**Passenger Cars/Pickups**		**Single Unit Trucks/Buses**		**Combination Trucks**	
		Mean	SD	Mean	SD	Mean	SD
Perfect	2-day	4.74	4.49	12.03	8.81	9.83	8.61
	1-week	3.40	2.36	9.00	6.84	9.12	7.61
	2-week	2.71	1.76	8.55	6.01	8.69	5.64
Current Year Bayes	2-day	5.84	4.76	11.03	9.42	11.40	8.42
	1-week	4.11	3.66	7.90	5.86	9.40	8.03
	2-week	3.23	3.00	7.55	4.87	8.13	5.05
Preceding Year Bayes	2-day	8.44	8.42	29.26	27.40	35.53	32.47
	1-week	6.22	4.46	13.00	10.58	15.31	11.63
	2-week	6.19	4.15	11.41	9.98	13.07	9.77
2-year-old Historic Bayes	2-day	13.05	11.80	45.16	32.79	71.05	59.56
	1-week	6.58	5.49	17.89	14.72	17.69	14.27
	2-week	5.96	4.97	13.65	12.67	14.09	9.68
Unadjusted	2-day	14.43	10.10	26.40	18.21	27.40	15.91
	1-week	13.77	8.51	17.16	10.62	10.77	9.50
	2-week	13.57	8.11	15.35	9.05	8.21	6.30

Figures 4.1-a -- 4.1-c are such plots for site 6251 in 1994. Figure 4.1-a are the estimator performance summaries for the Passenger Cars/Pickups category using 2-day, 1-week and 2-week samples. Figure 4.1-b is for the Single Unit Trucks and Buses and Figure 4.1-c is for the Combination Trucks category. The five estimators including three "Bayes" estimators, i.e., the "current year Bayes" estimator, the "preceding year Bayes" estimator and the "historic year Bayes" estimator, the "perfect" estimator and the "unadjusted" estimator. The horizontal dashed line represents the 95 percentile. Plots for other sites are attached in Appendix A. Table 4.1 gives the quantitative summaries of the estimator performance in terms of the mean estimation error and the standard deviation of estimation error for site 6251. All table entries are percentages.

From these two summaries, we have the following conclusion,

1. Comparing among the three Bayes estimators, the current year estimator has the best performance for all the three categories. A confidence level frequently used by traffic engineers and other professionals in the estimation is the 95 percent. Therefore, besides the mean error reported, a 95 percentile of estimation errors was also used in this study. And 95 percent of all the estimates were within 9.2% to 15.3% of the AADT estimates obtained by AASHTO Guidelines for Passenger Cars/Pickups depending on the size of the sample. And for the second count category, the estimation error is within 17.3% to 29.8%, again depending on 2-day, one week or two week samples. The last category has the highest level of volume variation, and therefore, the highest error of estimation. The 95 percentile APE is ranged from 18.2% to 28.2%. The Bayes estimator using the preceding year factor outperformed the 2-year-old historic one for all the categories with different sampling plans. Thus, when using Bayesian method to assign short count site to factor group and then adjust count samples to obtain classified MDT estimates, it is better to use the current year adjustment factors whenever possible. Since it is not feasible to obtain the current year factors, especially for the forecasting and estimation purpose, use of preceding year Bayes estimator is always recommended to the historic year one. For most cases, the current-year factors for any calendar year cannot be developed until after

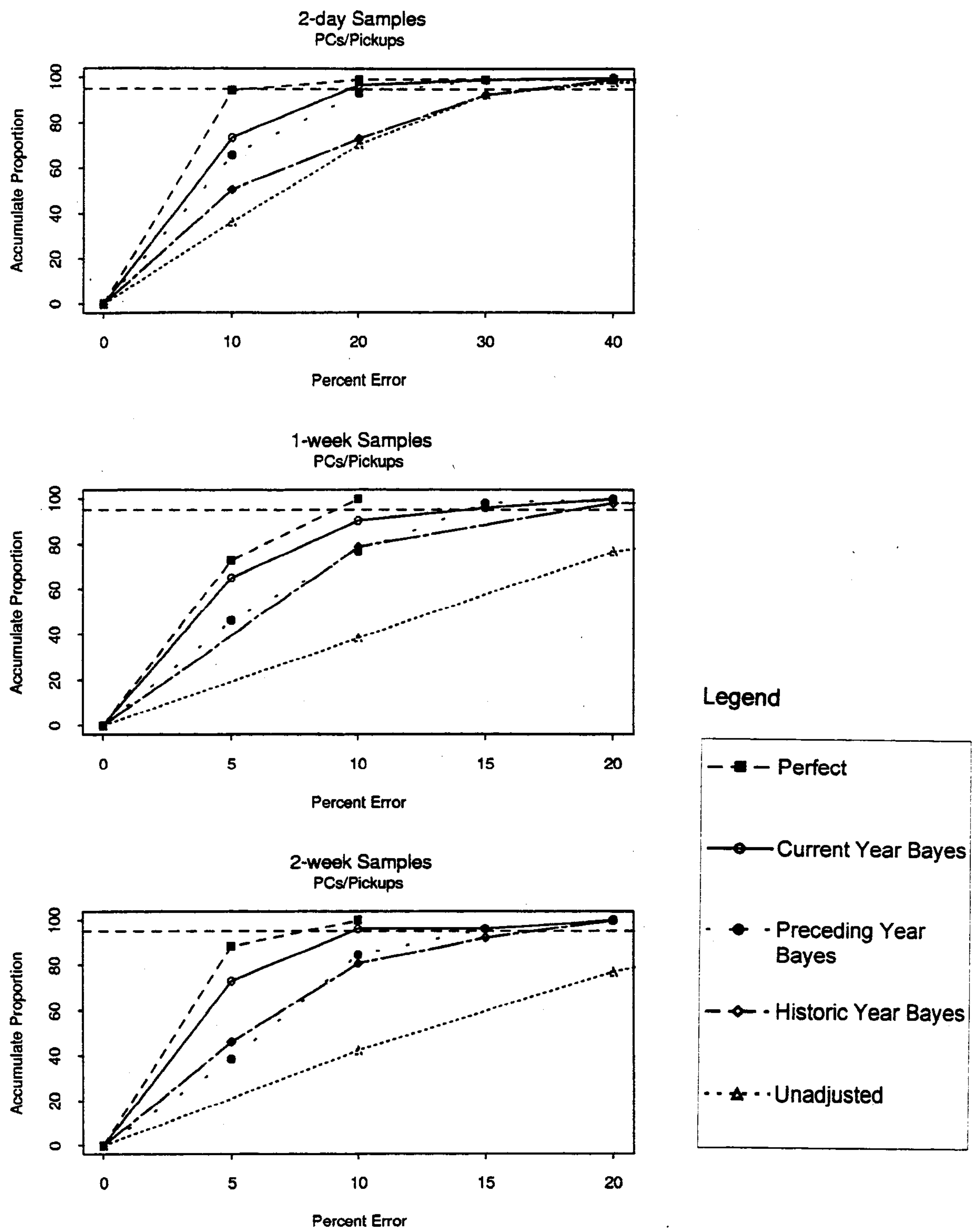


Figure 4.1-a Plot of the Estimator Performance Summaries for Site 6251 (94) PCs/Pickups

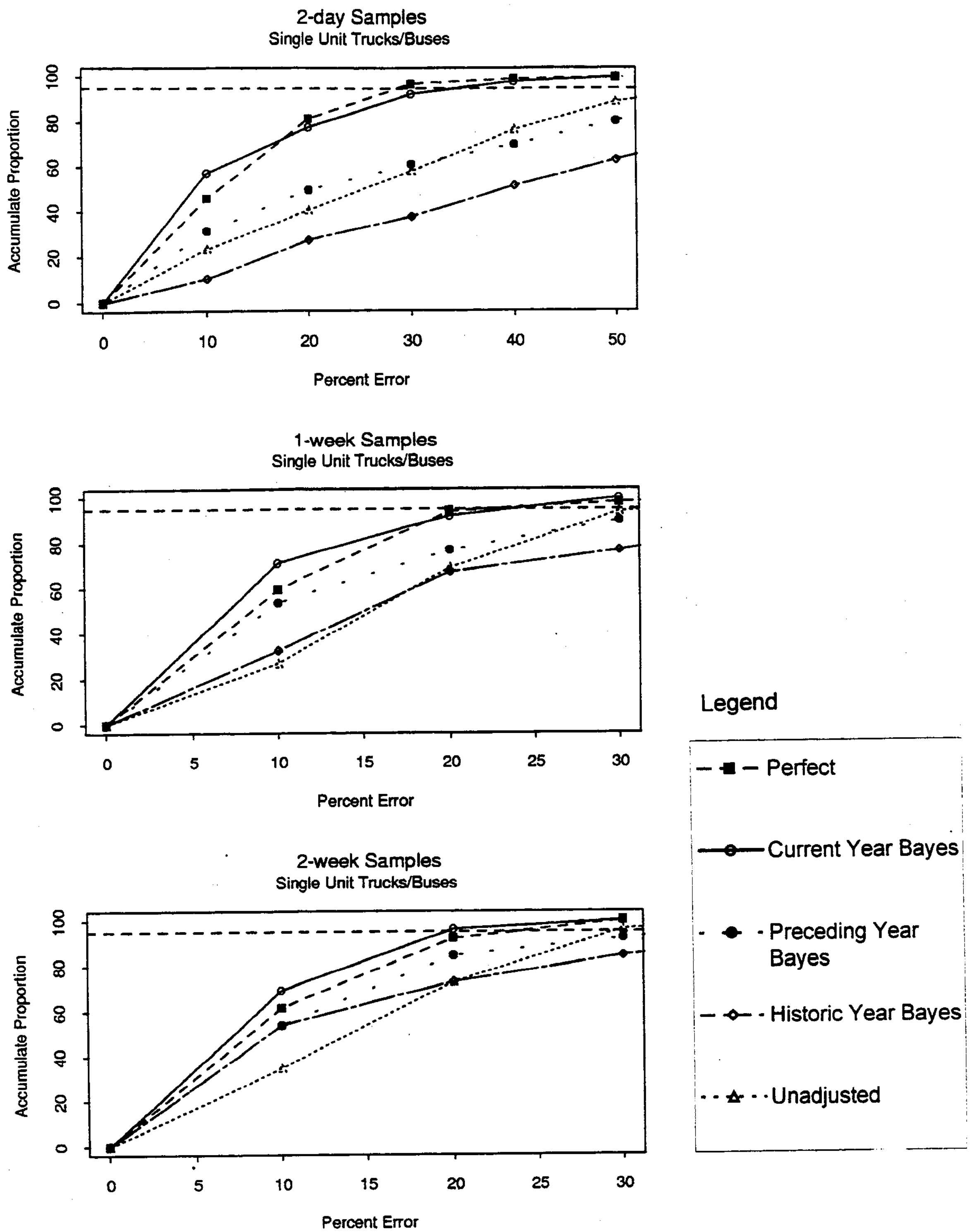


Figure 4.1-b Plot of the Estimator Performance Summaries for Site 6251 (94)
Single Unit Trucks/Buses

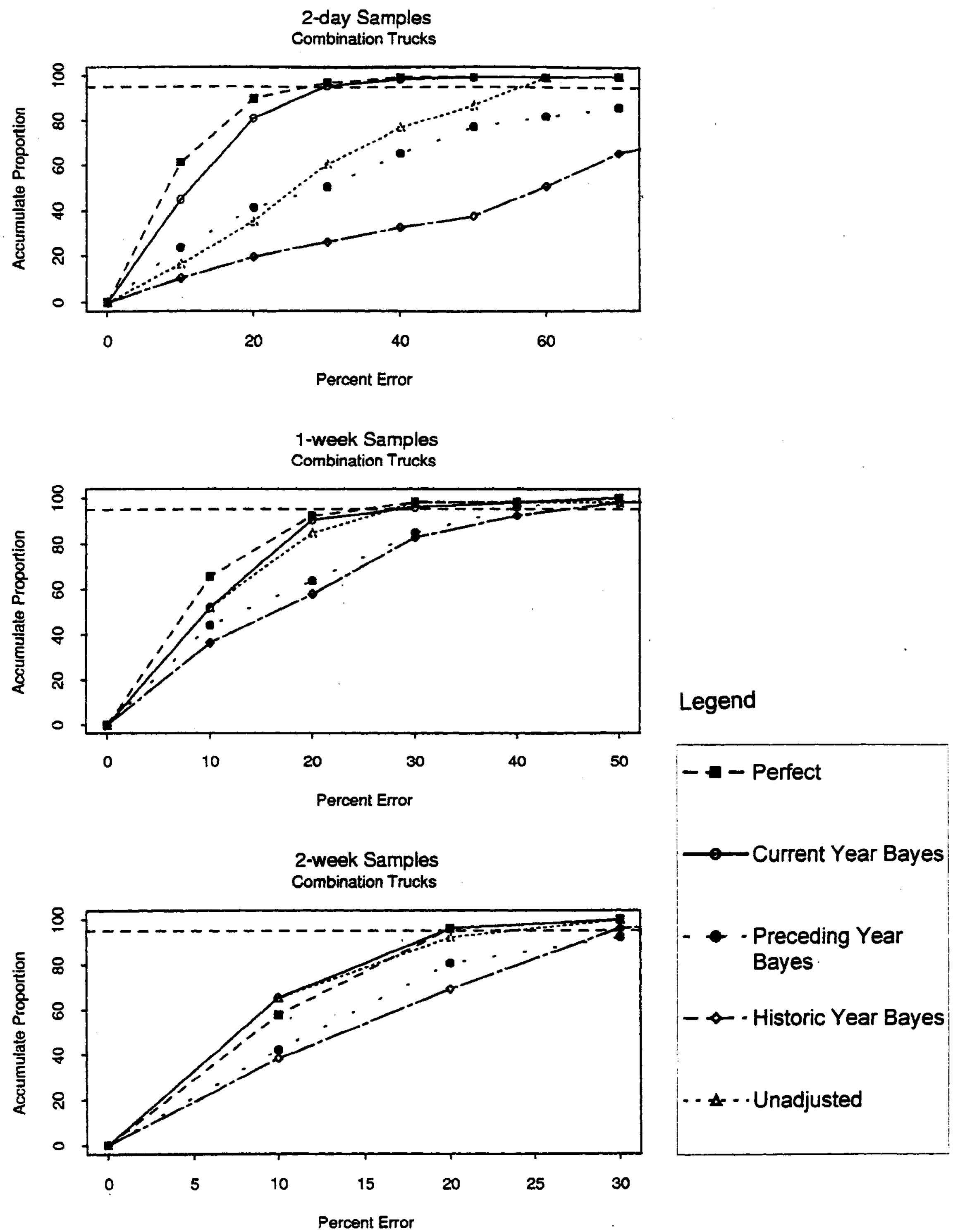


Figure 4.1-c Plot of the Estimator Performance Summaries for Site 6251 (94) Combination Trucks

the calendar year is over, thus the use of the current year factors is much limited and thus will not involve any further discussion.

2. For each count category in our study, as sample size increases, the performance of all the estimators increases substantially with the estimation errors almost halved from 2-day samples to 2-week samples. Therefore, if one wants to come up with some good estimates using arbitrary samples, one needs to pay for the sample size of at least one week sample, and this will bring the APE down to 14.5%,31.4% and 32.5% from 25.2%,83.1% and 100.5% for the three categories, 95 percent of time using the preceding year Bayes estimator.

3. To compare the results in Table 4.1 to those reported in other studies, it recalled that Hallenbeck reported that the mean estimation error for Bin 1 ranged from 6% to 9% for 1 to 3 weekday samples and 9% to 23% for Bin 3 and Bin 4. But what he used was the "best" factoring alternative where there is no site association errors and thus only the errors due to day-to-day variation was reported, as in our "perfect" case. Also recalled that Thomas, Sharma and Liu reported, when using 48 hour AVC sample counts to estimate truck AADT, the PB95 values ranged from 7.8% to 23.4% under the "ideal" scenario, and the mean estimation errors ranged from 10.7% to 13.5% and the standard deviation ranged from 8.3% to 15.4% under the "less-than-ideal" scenario.

4. Due to the computation method, the "perfect" estimates can be considered as the best MDT estimates we could have in general, and the second best estimates shown in our plot and table is the current year Bayes estimates. The results of the comparison of other Bayes estimators to the "unadjusted" estimator are mixed. For the Passenger Cars/Pickups category, the preceding year Bayes estimator worked much better than the unadjusted estimator, with the estimation error about 30% smaller during 95 percent of time for 2-day samples. When we move to the Single Unit Trucks/Buses, the two estimators have almost equivalent performance. But when estimating MDT for larger trucks, which has the highest

level of volume variation, the "unadjusted" estimates work better than the preceding year Bayes estimates using arbitrary sample counts. For all the test data available in this study, similar results were found and the "unadjusted" estimates of MDT of Combination Trucks work as good as, if not better than the Bayes estimates. Since 2-day samples were widely used in the practice of estimating truck volume, while the estimation error for the truck traffic ranged from 29% to 35% for arbitrary 2-day samples, which is large, we need to think of ways to get better estimates.

4.3 "Optimal" Sampling Design

In Chapter 3, we discussed how to use Bayesian decision methods to determine an optimal assignment of a highway site to a factor group, given a sample of daily classification traffic counts from that site. The optimal assignment is achieved by introducing the "0-1" loss function which assigns a uniform penalty for misclassifying the site, and no penalty for a correct classification. This led to the intuitively appealing decision rule of simply assigning the site to the factor group with the highest posterior probability, or equivalently, selecting the monthly and day-of-week adjustment terms that the sample count most favors. Using the lognormal likelihood of daily classification traffic counts developed in Chapter 2, the formula for computing the posterior classification probabilities were derived as in (3.15-3.16), using the results of the model fitting to develop an empirical Bayes informative prior for the site autocorrelation coefficient matrix and noise covariance matrix parameters. In this Chapter, we further apply this procedure to finally develop an algorithm to calculate the Bayes estimates of classified MDT. And under a squared error loss function, the MDT for multiple count categories is just the summation of the expected MDT given the sample and the factor group membership weighing over the posterior classification probabilities. Again by using the lognormal model and the empirical priors for those parameters, the formula for computing Bayes estimates of classified MDT is shown in equation (4.12). Since the application of these formulas require that a sample of daily classification traffic counts of the highway site be available, but it may very well be that classification traffic counts on some days are more informative than counts made on other days, as was shown by the results in Table 3.2--

3.4. In this section, we would like to investigate the problem of optimal samples. Since the normal practice in the state of Minnesota is to collect 2 day samples, we will concentrate our interest in designing sampling plans for 2-day samples which are more likely to lead to correct assignment decision and therefore more accurate Bayes estimates of classified MDT.

In Davis [26], a heuristic solution was achieved to a nonlinear knapsack problem to design the optimal sampling plan for the univariate estimation problem, which only involves three factor groups. Since we have more members in the factor group and the sample size we are interested is 2-day samples, the heuristic solution is not applicable in our optimization problem.

From the analysis in Chapter 2, we know that classification traffic counts tend to show some seasonal and day-of-week variations, and it is not difficult to imagine that some 2-day samples will tend to produce larger estimation errors than other samples. Thus, plotting the absolute percent errors, as was introduced in equation (4.13), for each sample should tell us the variation of the estimation errors in terms of the sampling time. Therefore, we plotted the percent errors of the preceding year Bayes estimates of classified MDT using all the available 2-day samples, and were able to see the weekly trend of the estimation errors and identify the samples which incur large estimation errors. These were done for all the 1994 WIM sites. And it is concluded that the "optimal" sampling time in a week is to sample on the combinations of Tuesday, Wednesday and Thursday (i.e.,Tue-Wed, Wed-Thur, Thur-Tue) but not on the other days. After removing the day-of-week variation, we then plotted the estimation error again to check the monthly trend and it was found that for most sites in Minnesota in the year of 1994, in order to obtain more accurate estimates of classified MDT volume, it is better not to take samples in the month of January, February, November and December. To compare this finding with those reported in other studies, recall that in Hallenbeck's report, the best sampling time is for individual weekdays or combinations of weekdays, and the weekdays are defined as Tuesday, Wednesday and Thursday. During the course of designing "optimal" samples, besides the visual check method mentioned above, we also computed the estimation errors with different sampling designs for all the dataset available to this study. Table 4.2 summarizes the improvement of each sampling method in terms of mean estimation errors when compared to the arbitrary 2 day samples. Each table entry is calculated by letting the decrease of the mean estimation error for the corresponding sampling plan in terms of the mean estimation error using arbitrary samples divided by the latter. For example, the mean estimation error for Single Unit Trucks using only the weekday samples is 14.06 while it is 40.97 when using arbitrary 2-day samples and thus the improvement is 65.7%.

Table 4.2 Performance Improvement Using Different Sampling Designs

	Performance Improvement		
Sampling Design	**Passenger Cars/Pickups**	**Single Unit Trucks/Buses**	**Combination Trucks**
Throw out Mondays,Sundays& Saturdays	-12.4	52.6	59.9
Throw out Fridays	26.2	56.5	65.7
Throw out Winter	38.9	59.4	71.0

The interpretation of the table is, by throwing out the Monday, Saturday and Sunday counts, the estimation was, on the average, 59.9% closer to the AADT obtained using AASHTO Guidelines for Combination Trucks compared to using arbitrary 2-day samples. And if we throw out Friday samples in addition, it will be 65.7% closer to the AADT. And it is expected to be 71% closer if we restrict our samples on the combinations of weekdays excluding in the month of January, February, November and December. Notice that for the Passenger Cars and Pickups, unlike for the trucks, the estimation accuracy hasn't been improved when removing the Mondays, Saturdays and Sundays samples.

Thus by plotting the percent errors, we were able to detect those samples that incur large estimation errors and therefore could come up with "optimal" sampling design. That is, for the sites in the state of Minnesota, the best sampling time for 2-day samples is to sample on weekdays of March to October. Using this sampling design, we obtain the "optimal" Bayes estimates of classified MDT, and the plot in Figure 4.2 is the performance summary for preceding year Bayes estimators using "optimal" 2-day samples, arbitrary 2-day samples, as well as "perfect" and "unadjusted" estimators using "optimal" 2-day samples for site 6251. Plots for other sites available for this study are attached in Appendix A. From Figure 4.2, we can see that using the "optimal" samples, for the Passenger Cars and Pickups, we are able to reduce the 95 percentile estimation error to 12.2% using "preceding year Bayes"estimator and it is very close to that of the ideal situation, which is about 9.5% shown by the "perfect" estimator. A much larger estimation error (about 26%) results if we do not use any adjustment. For the Single Unit Trucks and Buses, the "Bayes" estimator works even a little better than the "ideal" case and the 95 percentile estimation error for "Bayes" estimator, "perfect" estimator and "unadjusted" one are about 24.7%, 28.7% and 52.2%, respectively. For the Combination Trucks, the 95 percentile estimation error for "Bayes" estimator, "perfect" estimator and "unadjusted" estimator are about 21.8%, 24.7% and 49.8%, respectively. Therefore, using the "optimal" samples, the estimation errors could be reduced more than half for the trucks at site 6251. Table 4.3 is a summary, across all sites available in this study, of the 95 percentile estimation errors for Bayes estimators using different sampling plans and the other two estimators. Using the "optimal" 2-day samples, the estimation error of MDT for Single Unit Trucks/Buses is within 22.8% for 95 percent of all the

Table 4.3 Estimator Performance Summary

Estimation Errors For 95 Percent of Samples			
	Passenger Cars/Pickups	**Single Unit Trucks/Buses**	**Combination Trucks**
"Perfect" Estimator	12.32	23.53	26.24
Bayes Estimator using "optimal" samples	15.19	22.8	25.50
Bayes Estimator using "arbitrary" samples	27.62	76.27	125.75
"Unadjusted" Estimator	24.56	45.02	60.58

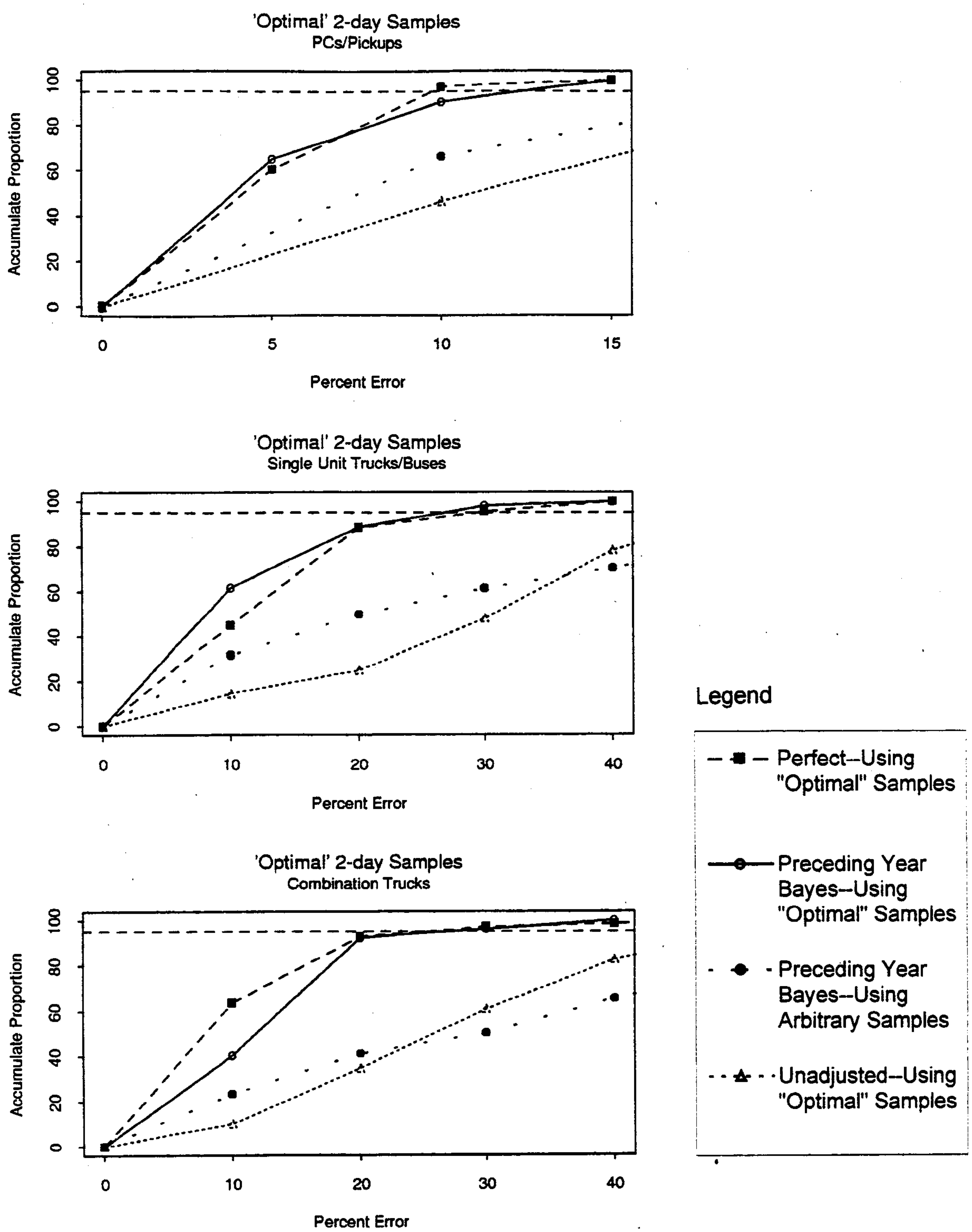


Figure 4.2 Plot of the Estimator Performance Summaries for Site 6251 (94) "Optimal 2-day Samples"

samples for all WIM sites in 1994 versa 76.27% estimation error for "arbitrary" 2-day samples and 45.02% if just averaging the "optimal" samples. The greatest benefit was seen on the Combination Trucks category, the estimation error drops dramatically from within 125.75% using "arbitrary" 2-day samples and 60.58% without using any adjustment to 25.50% using "optimal" 2-day samples, 95 percent of time.

In order to further investigate the performance among the combinations of weekday samples, Table 4.4 summarizes the average percent errors of different 2-day samples (i.e., Tue-Wed, Wed-Thur and Thur-Tue) for all the sites in 1994. For the PassengerCars and Pickups count category, it is safe to sample weekday samples anytime excepts January, February and December. For the Single Unit Trucks/Buses and Combination Trucks, weekday samples in January, February, November and December tend to have larger estimation errors. From the table, we can also identify the time to collect 2-day samples. For example, the Tue-Wed and Thur-Tue samples in the month of May and June, the Wed-Thur and Thur-Tue samples in August and September as well as the weekday samples in the month of October tend to produce better truck MDT estimates for most sites available to this study.

4.3 Conclusion

In conclusion, the effect of factor group assignment error can be seen clearly in Table 4.3. The "unadjusted" estimator ignored information concerning factor group membership, and thus generated large estimation errors for classified MDT. In contrast, the Bayes estimator uses the membership information and when combined with the "optimal" sampling design we have come up with, it provides better estimates of classified MDT. Particularly encouraging is the fact that for trucks, the estimation error using Bayes methodology is expected to be within 22.8% and 25.50% at 95 percent of time for Single Unit Trucks/Buses and Combination Trucks with 2-day samples.

Table 4.4 Mean Estimation Error of Weekday Samples

	Passenger Cars/Pickups			Single Unit Trucks/Buses			Combination Trucks		
Month	Tue-Wed	Wed-Thur	Thur-Tue	Tue-Wed	Wed-Thur	Thur-Tue	Tue-Wed	Wed-Thur	Thur-Tue
Jan	23.00	10.68	11.98	24.03	16.72	8.03	33.24	11.56	19.43
Feb	15.02	12.35	13.18	13.60	15.02	16.34	15.03	22.64	19.07
March	9.14	9.28	7.12	16.87	15.60	12.67	11.72	20.16	12.62
April	11.81	6.40	8.74	13.22	12.35	13.20	19.41	15.56	15.65
May	5.59	4.78	7.26	8.42	7.21	11.64	9.72	14.34	11.05
June	7.69	6.04	5.41	7.81	10.59	9.23	14.59	15.36	12.37
July	7.68	6.89	8.55	10.75	10.12	11.45	10.14	13.29	11.80
Aug	8.91	7.26	8.40	9.08	9.00	10.77	8.74	8.03	8.40
Sept	8.28	5.80	5.35	14.30	11.58	8.42	8.43	7.11	6.01
Oct	8.56	8.41	7.32	8.61	7.56	10.21	10.97	8.70	9.69
Nov	9.27	9.04	9.97	8.77	10.47	9.16	21.49	24.15	16.00
Dec	12.71	9.69	14.34	11.39	10.19	14.47	8.53	18.89	17.58

CHAPTER 5
SUMMARY AND CONCLUSIONS

5.1 Summary and Conclusions

In Chapter 1, we pointed out that classification MDT is one of the most important estimates used in modern highway and pavement design, but limitations in personnel and equipment have hindered the research into estimating MDT by vehicle class. And more, a number of states in the United States have expressed both a need as well as difficulty, in estimating classification MDT. In this project, we investigated the problem of obtaining accurate and reliable estimates of classification MDT using the prevalent factor group assignment method, via Bayesian Decision Theory. Our first objective was to establish a statistically accurate model to describe the daily classification traffic counts. Calibration and goodness of fit testing suggest that the lognormal regression model, coupled with a multivariate autoregressive model which accounts for serial correlation in the regression residuals, yields satisfactory fits to the one lane, one direction, three-category classification traffic counts collected by permanent WIM recorders in the State of Minnesota. Based on this model, we were able to estimate the monthly and day-of-week adjustment factors for each WIM dataset and therefore investigate the monthly and day-of-week trends existing in classification traffic counts. Our results showed that the monthly and day-of-week trends do exist in the classification counts and they are quite different from that of the passenger cars or vehicles in total. This suggests that use of class-specific monthly and day-of-week adjustment factors will improve the estimates of MDT by vehicle classifications.

Based on the statistical model developed in Chapter 2, we then tried to solve the factor group assignment problem for classification counts via a Bayesian approach. By assuming that the WIM sites obtained from LTPPP database from 1992 to 1994 are representatives of all the Minnesota highway sites for each year, we set up the factor group assignment problem in the Bayesian decision framework by adopting a "0-1" loss function, a noninformative prior for classification probabilities and empirical Bayes priors for the site specific parameters. Using the lognormal likelihood function, we were able to derive the formula to compute the posterior classification probabilities. Using a S-plus program, an empirical evaluation of the assignment problem using a 12-day sample counts from WIM sites showed that the algorithm worked quite well by assigning the sites to the factor groups having the same site number in another year.

We then proceeded with our third objective, i.e., to estimate the MDT by vehicle classification using Bayesian theory. By using a squared error loss function, the Bayes estimates of MDT is just the summation of the expected MDT given the sample and the factor group membership, weighted by the posterior probabilities. On average, it will be the estimator which is closer to the true MDT. We then investigated the impact of the adjustment, the sample size and different year seasonal factors on the accuracy of the estimates by comparing the performance of several estimators: the "Perfect" estimator, the "Current Year Bayes" estimator, the "Preceding Year Bayes" estimator, the " 2-year-old Historic Bayes" estimator and the "Unadjusted" estimator. As the sample size increases from 2-day to 2 weeks, the Bayes algorithm works much better, especially for larger trucks, and is able to cut the mean estimation error from 35.53% to 13.07% for the combination trucks. The use of the preceding year adjustment factors would be recommended over the 2-year old historic factors, if the current year factors are not applicable. Since the Bayesian approach uses the data information, and traffic counts on some days would be more informative than others, as was showed by the results in Chapter 3 and 4 , we then investigated the "optimal" sampling design in order to obtain more accurate estimates of MDT. It was found that for highway sites in Minnesota, the best time to sample 2-day counts is on weekdays (Tuesday--Thursday) in months of March to October. Using this "optimal" sampling design, Bayes estimation error of classification MDT is reduced substantially to 15.19%,22.80%,and 25.50% for the three categories 95 percent of time with preceding year factors.

5.2 Recommendations for Implementation

The methodology described in this report is applicable in any jurisdiction for which traffic estimation is accomplished using (1) a small number of permanent classifications counters, such WIM equipment and (2) short classification counts on the majority of road segments. Using daily count data from the permanent count stations, estimates of the seasonal and day-of-week adjustments can be computed using standard linear regression methods applied to the natural logarithms of the counts. Software implementing linear regression is available in most comprehensive statistical packages, including Minitab, S-Plus, SAS, SPS and BMDP. Estimates of the permanent site's autoregressive parameters and noise covariance matrix can also be carried out using regression methods. Once computed, these estimates can be stored in text files for use by the short-count estimation routine. The computation and updating of these estimates would probably be the responsibility of the office which maintains and processes the permanent recorder data.

For this project, software implementing the Bayesian classification and MDT estimation methods was written in S-Plus, and listings of these routines have been included in Appendix B of this report. Thus any agency currently using S-Plus for its statistical analyses can implement our methods without delay. For those agencies not using S-Plus, the procedure could be implemented in almost any computer language, including Matlab, SAS, Fortran, C and Basic.

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APPENDIX A

ADDITIONAL RESULTS ON ESTIMATOR PERFORMANCE SUMMARIES

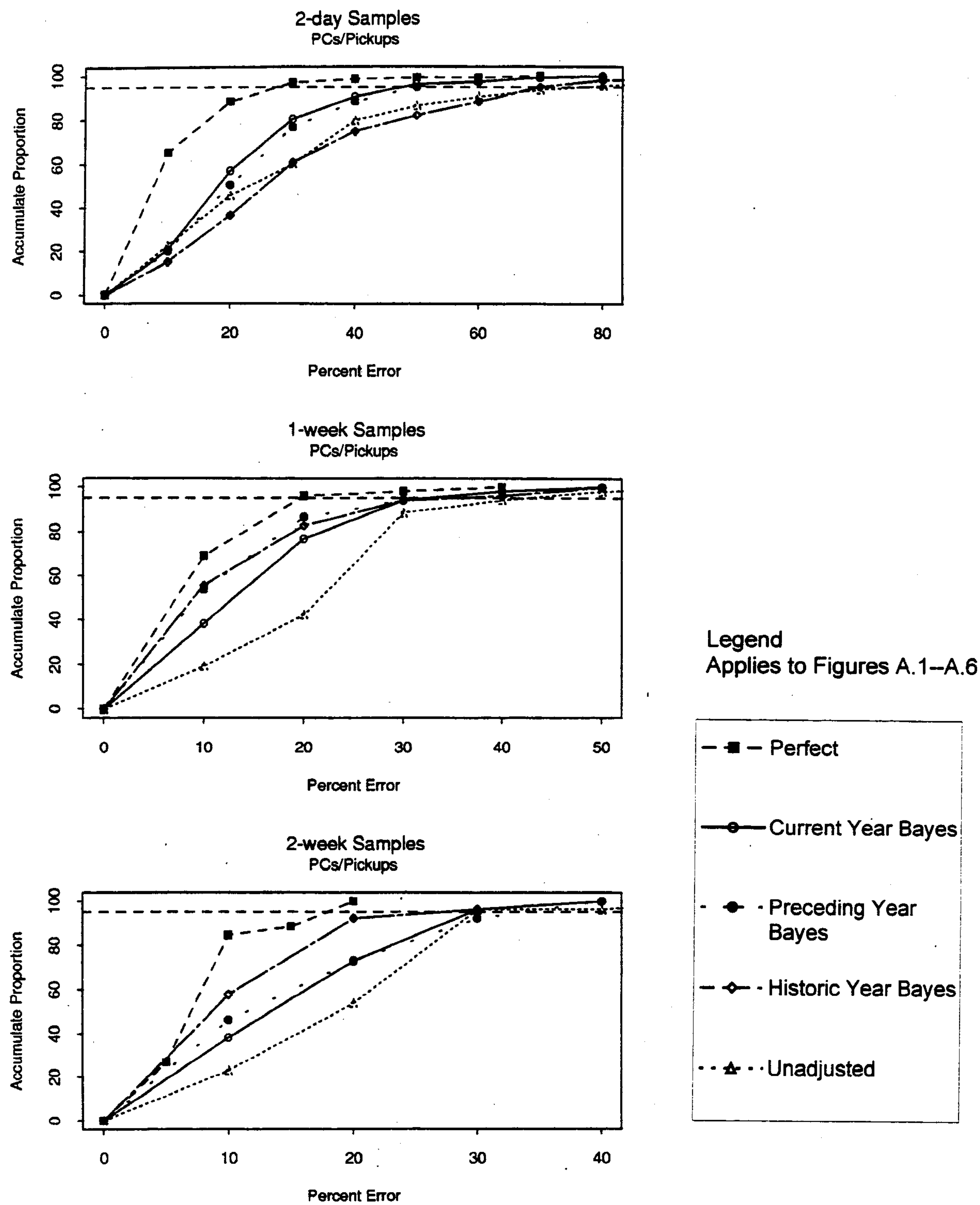


Figure A.1-a Plot of the Estimator Performance Summaries for Site 1019 (94) PCs/Pickups

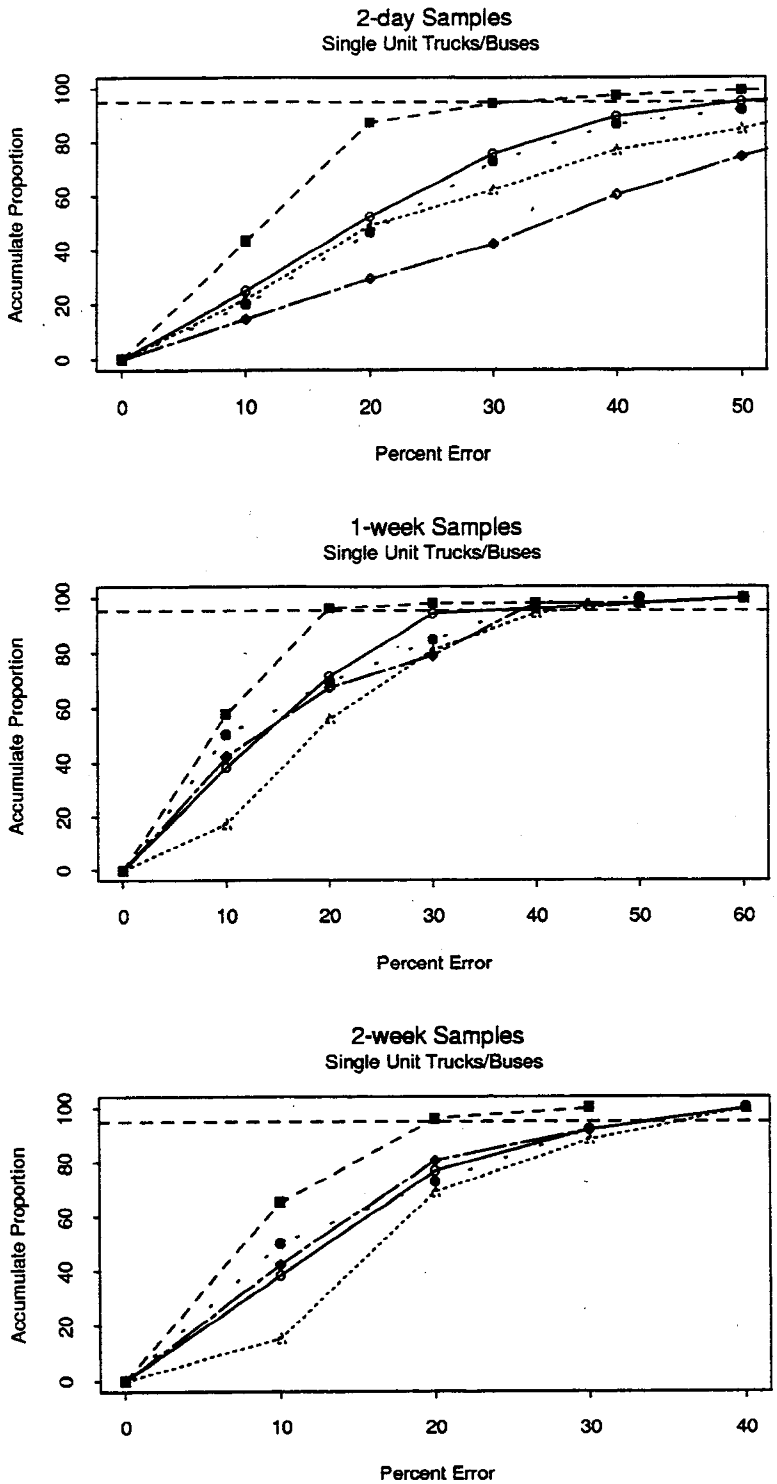
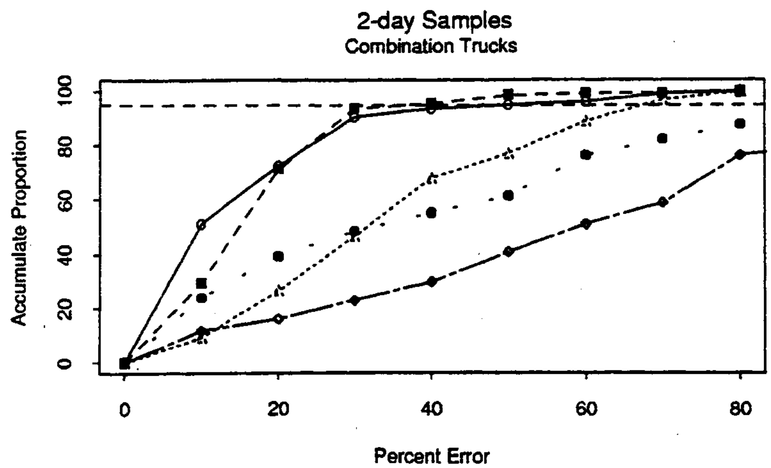
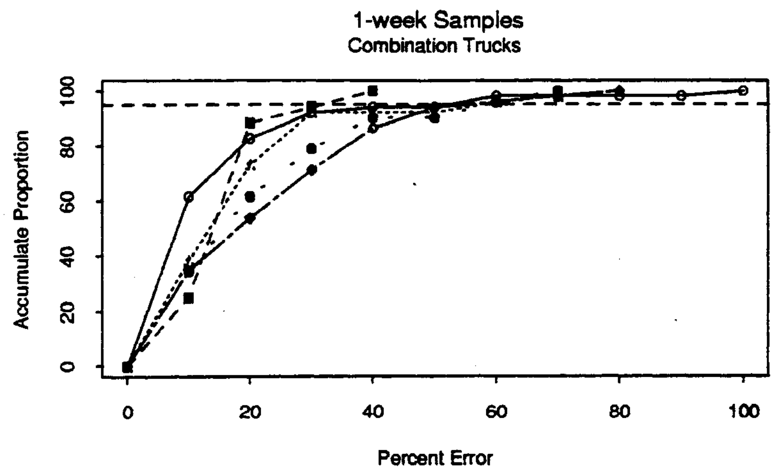


Figure A.1-b Plot of the Estimator Performance Summaries for Site 1019 (94) Single Unit Trucks/Buses





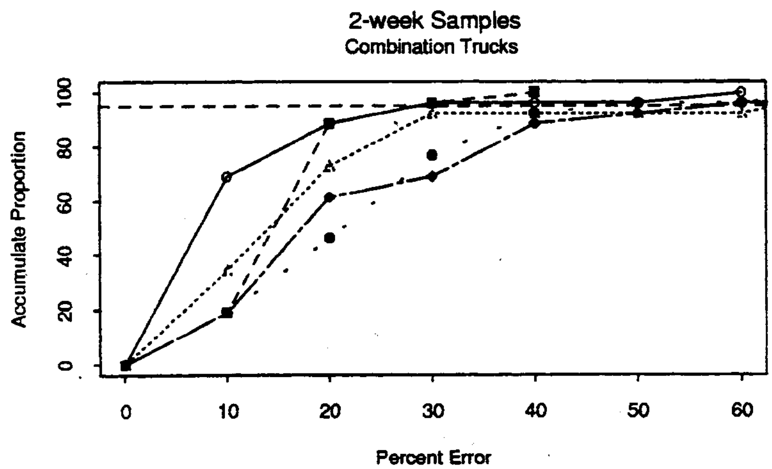


Figure A.1-c Plot of the Estimator Performance Summaries for Site 1019 (94) Combination Trucks

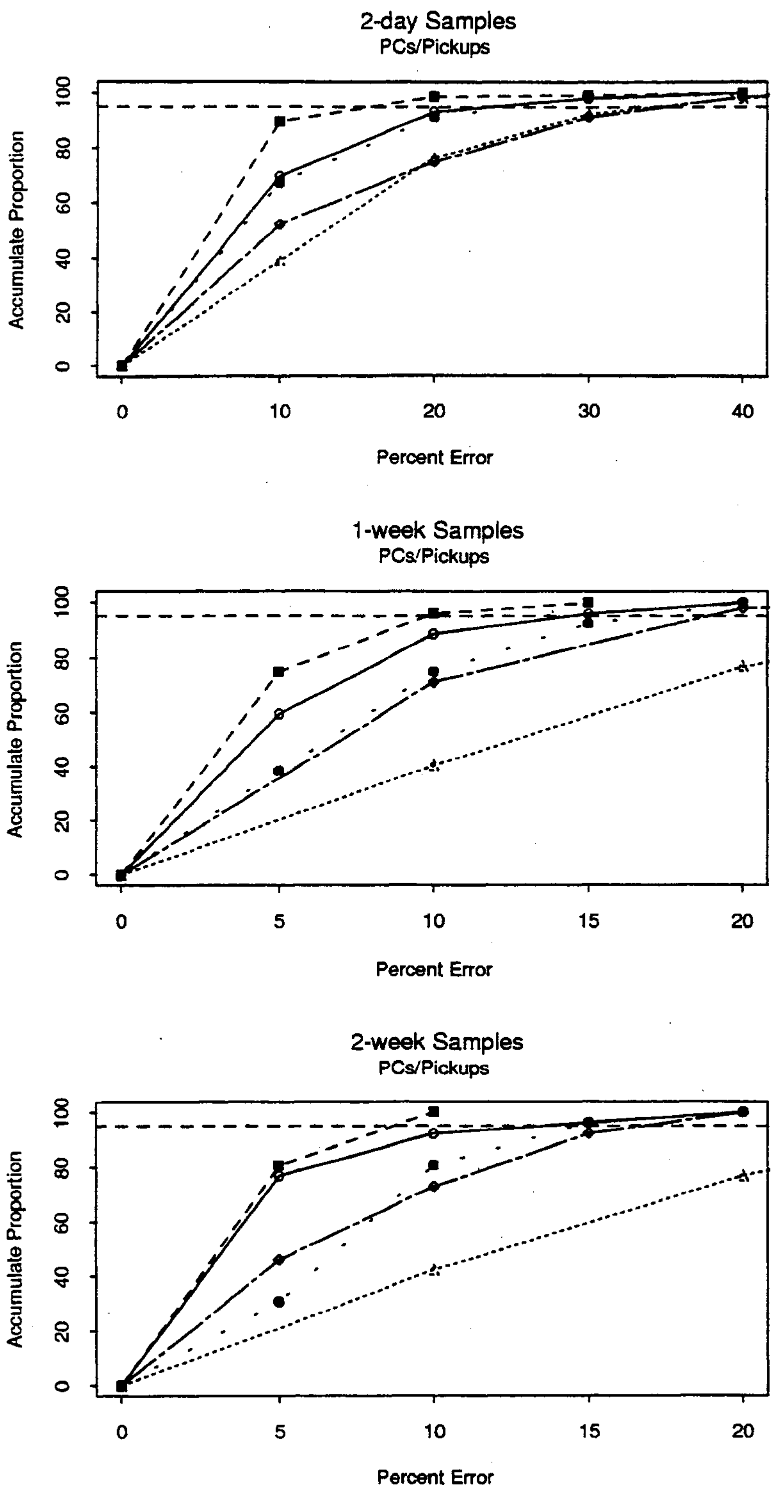


Figure A.2-a Plot of the Estimator Performance Summaries for Site 1023 (94) PCs/Pickups

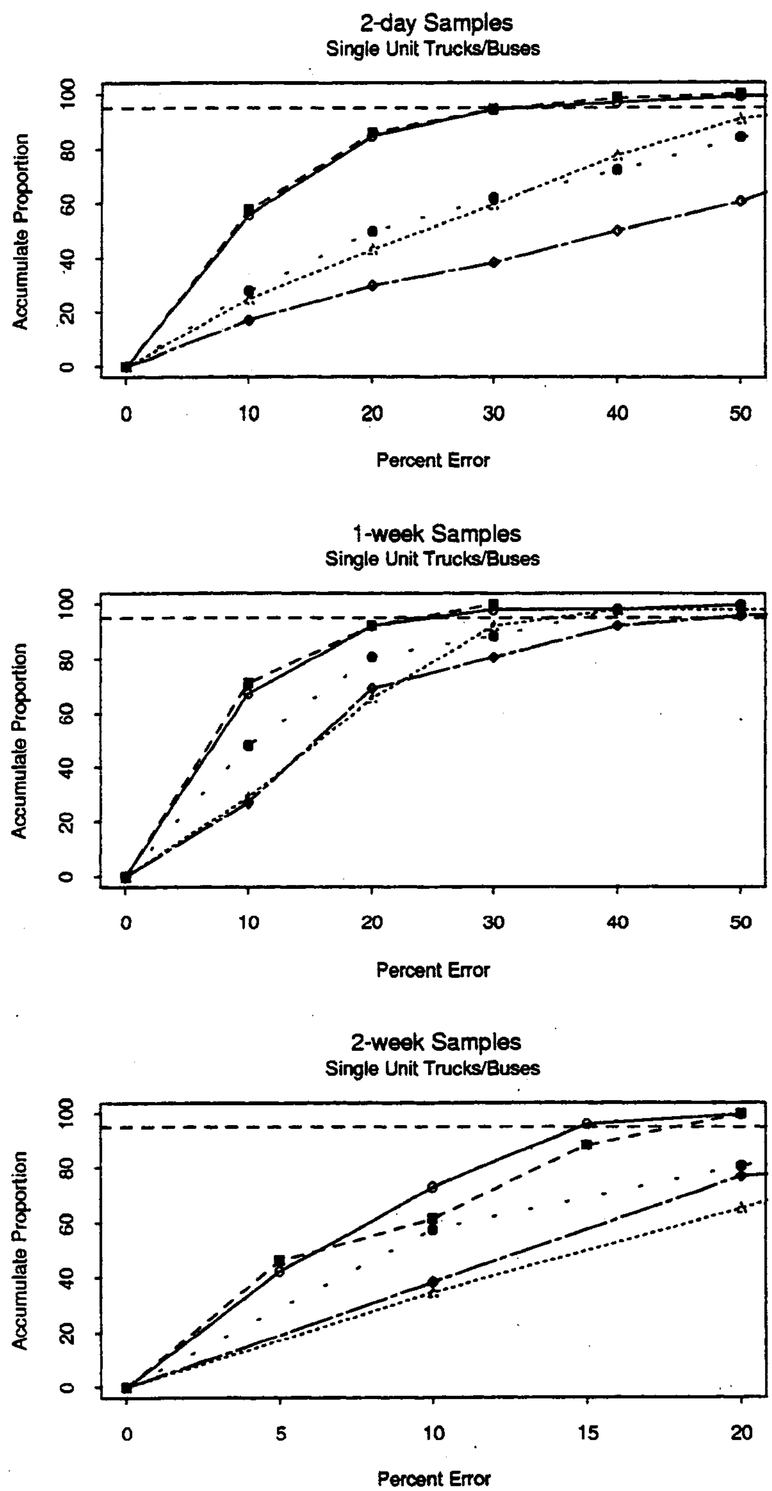


Figure A.2-b Plot of the Estimator Performance Summaries for Site 1023 (94) Single Unit Trucks/Buses

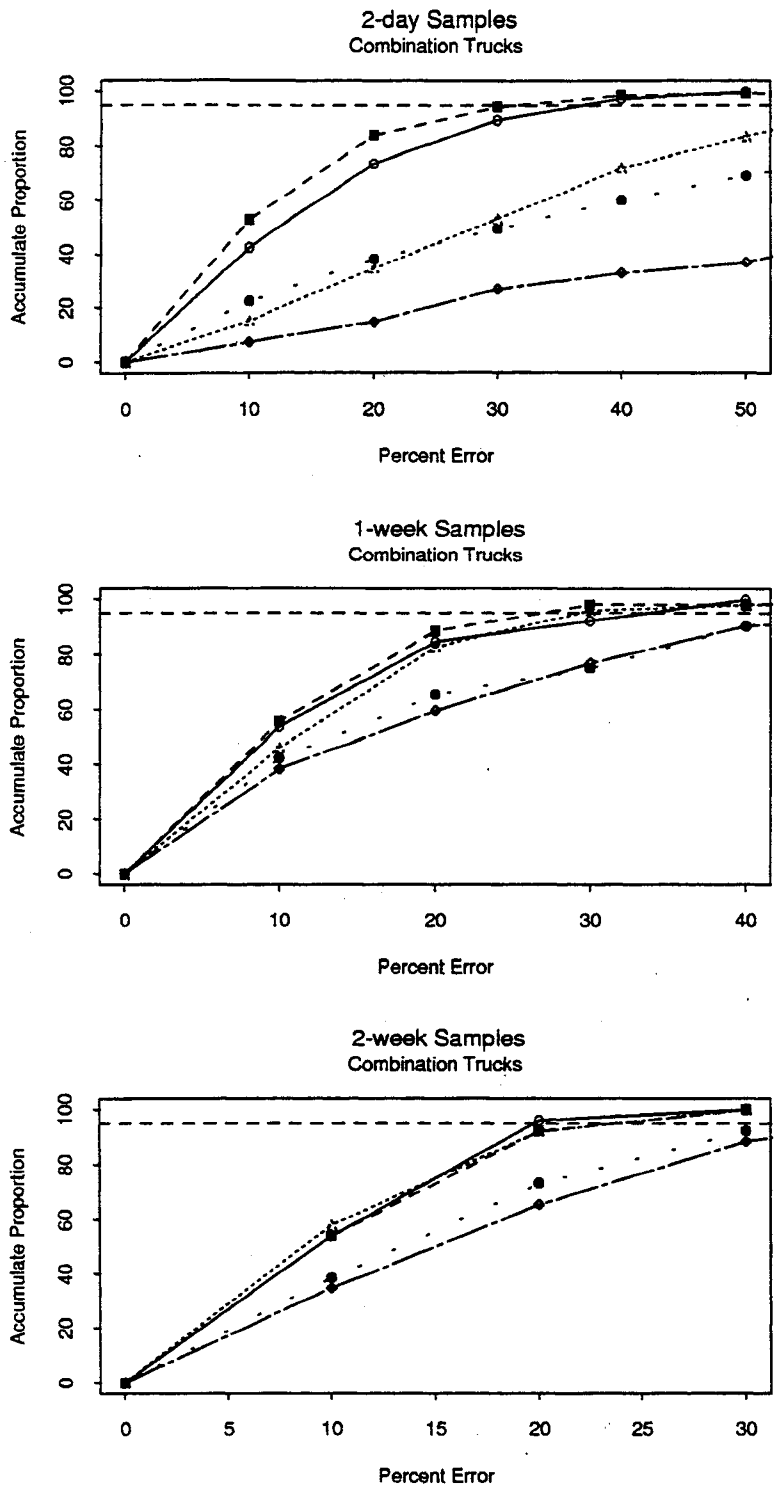


Figure A.2-c Plot of the Estimator Performance Summaries for Site 1023 (94) Combination Trucks

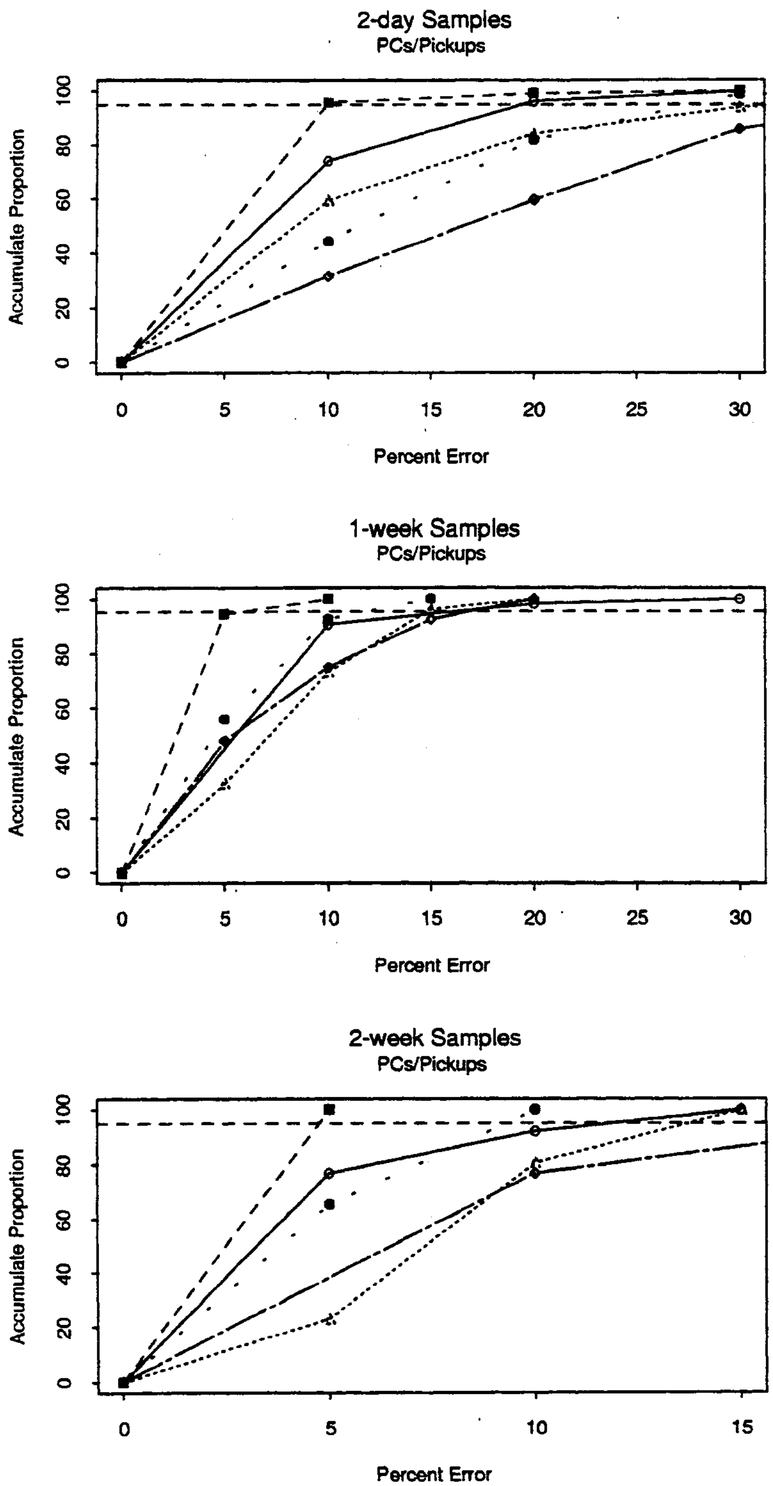
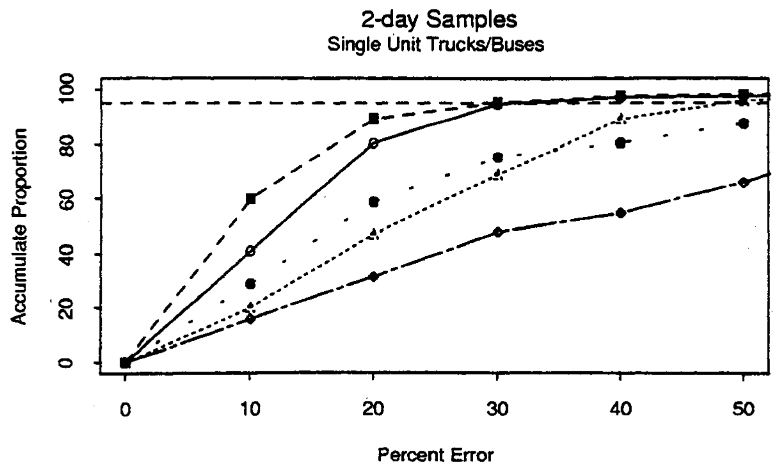
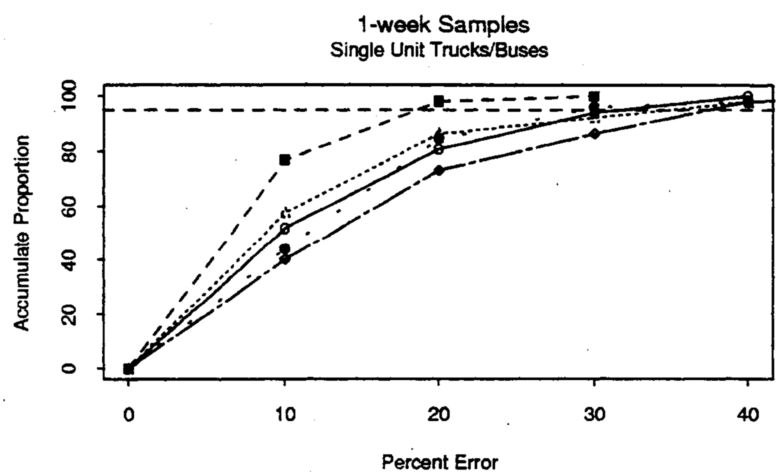


Figure A.3-a Plot of the Estimator Performance Summaries for Site 1029 (94) PCs/Pickups





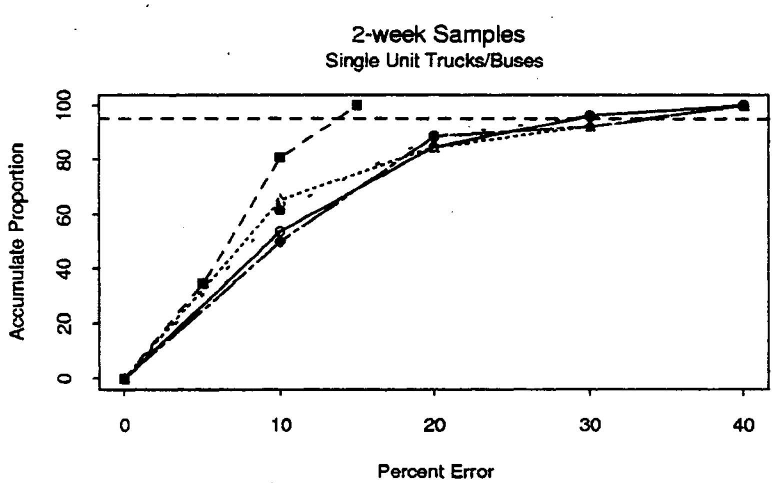


Figure A.3-b Plot of the Estimator Performance Summaries for Site 1029 (94)
Single Unit Trucks/Buses

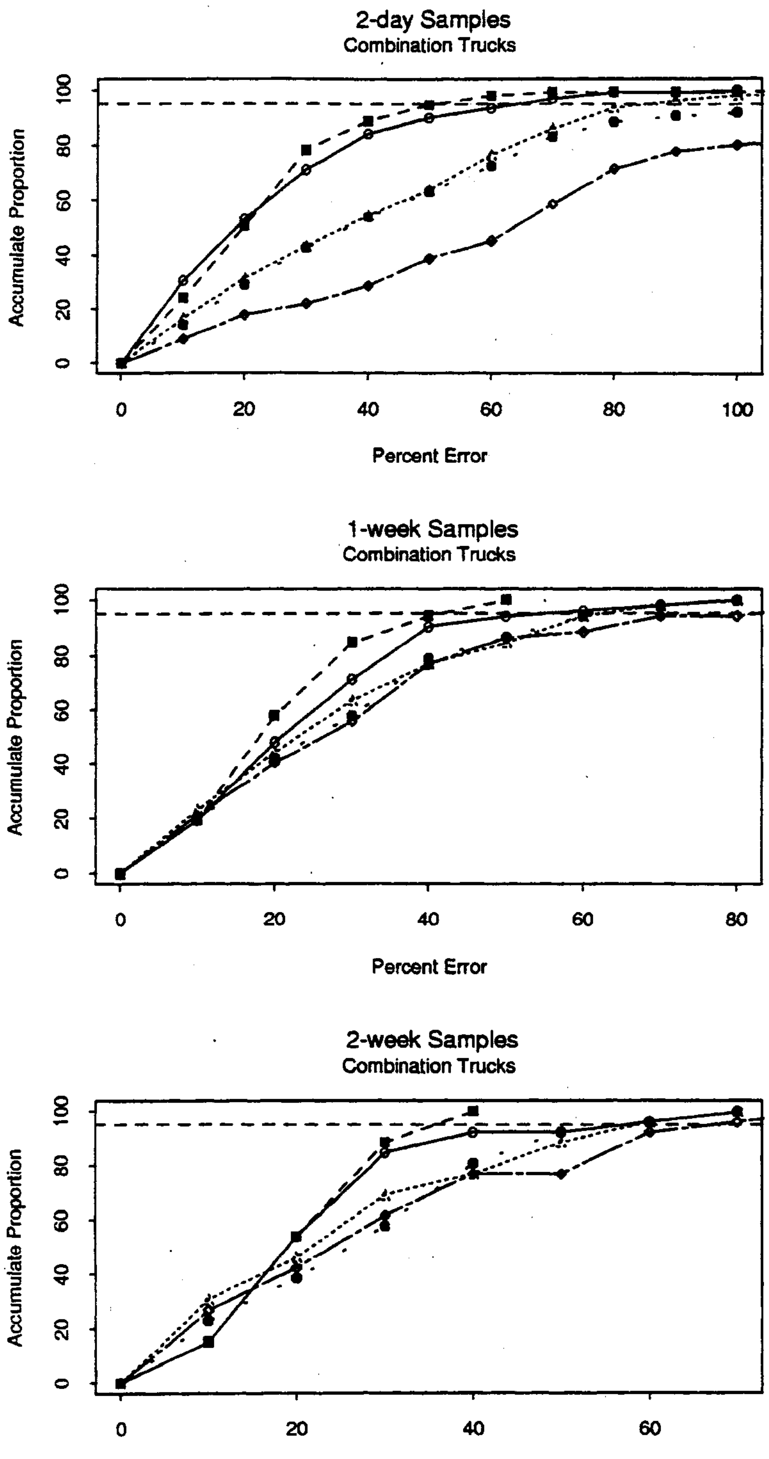
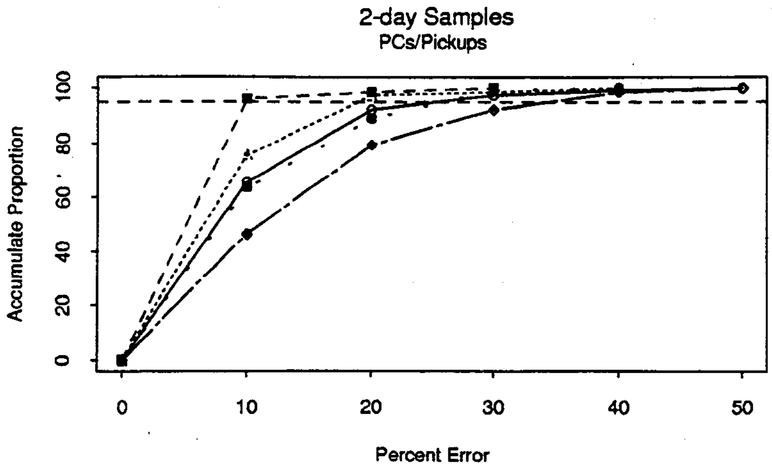
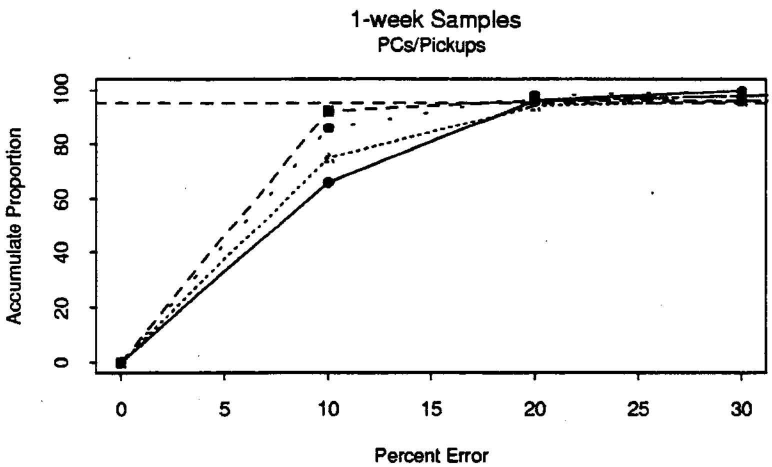


Figure A.3-c Plot of the Estimator Performance Summaries for Site 1029 (94) Combination Trucks





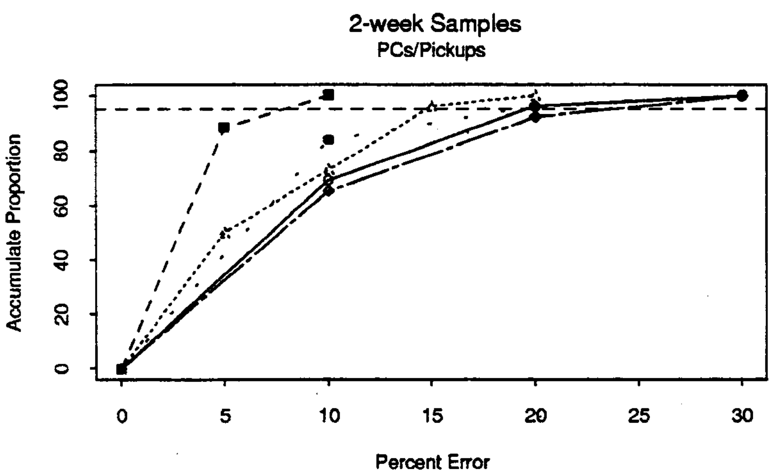


Figure A.4-a Plot of the Estimator Performance Summaries for Site 4033 (94) PCs/Pickups

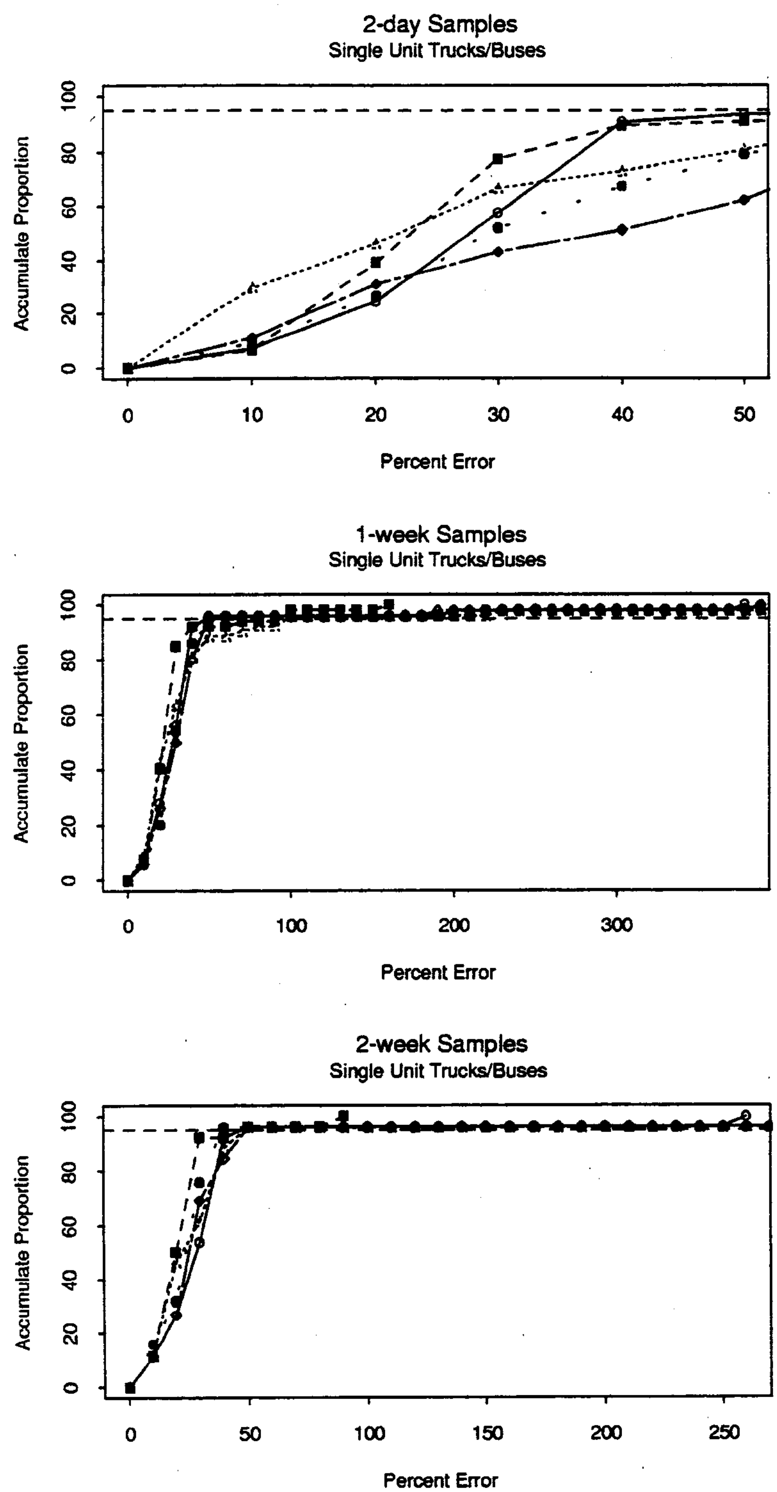
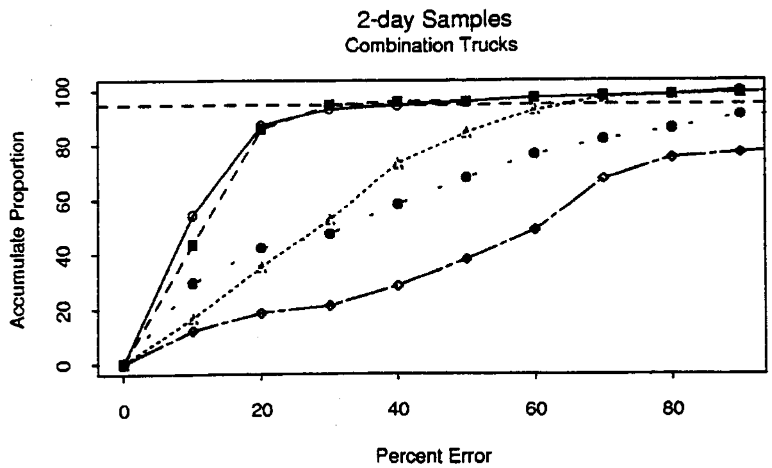
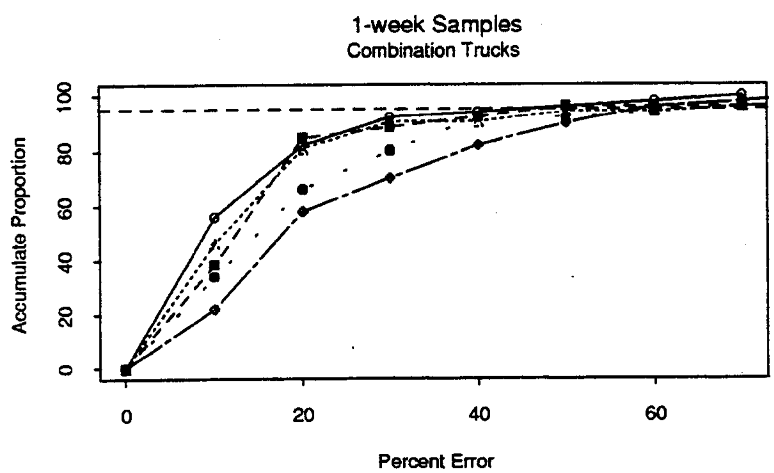


Figure A.4-b Plot of the Estimator Performance Summaries for Site 4033 (94) Single Unit Trucks/Buses





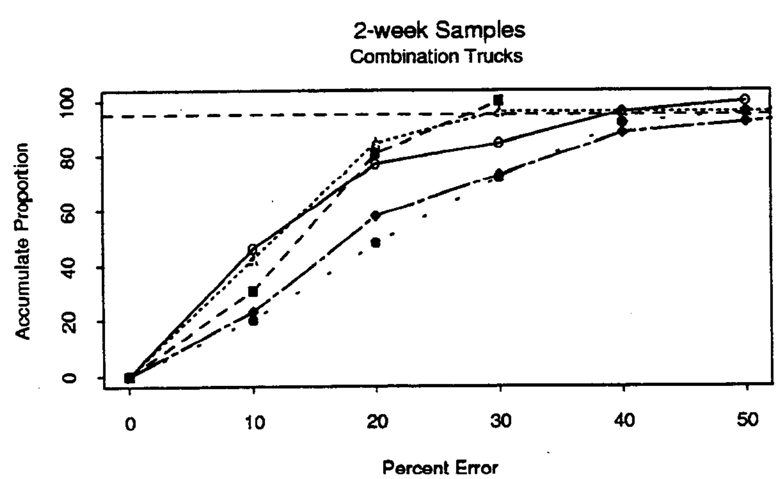


Figure A.4-c Plot of the Estimator Performance Summaries for Site 4033 (94) Combination Trucks

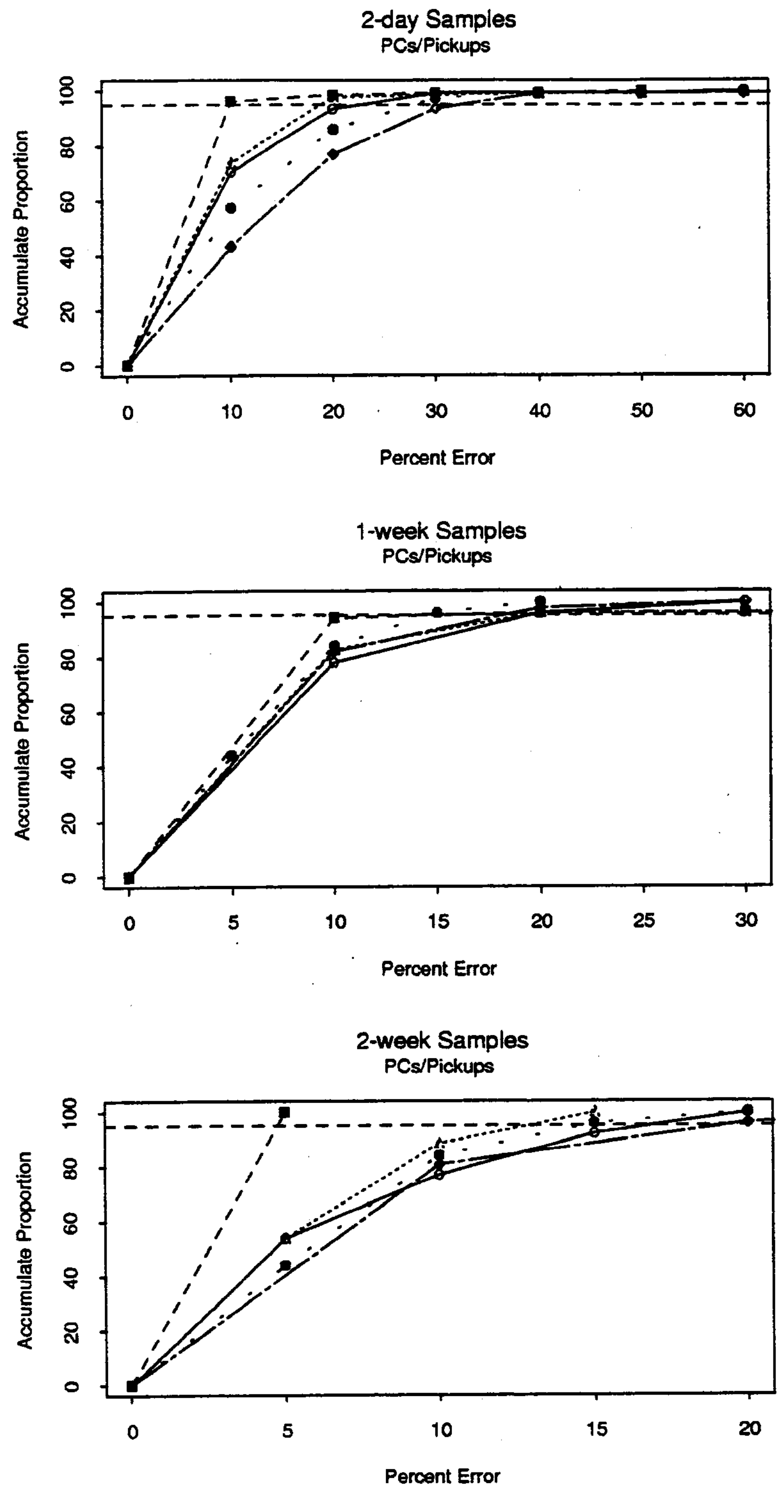


Figure A.5-a Plot of the Estimator Performance Summaries for Site 4037 (94) PCs/Pickups

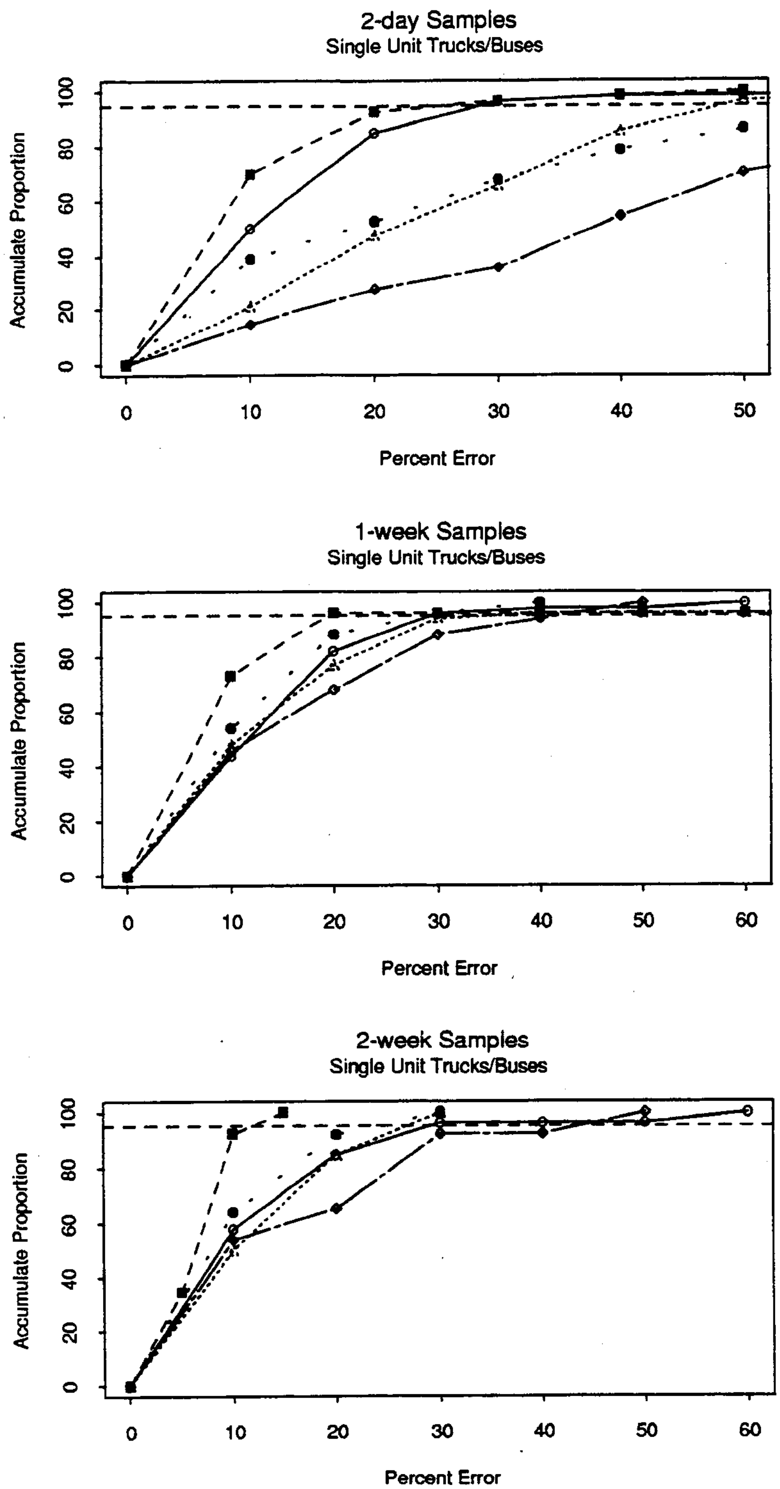


Figure A.5-b Plot of the Estimator Performance Summaries for Site 4037 (94) Single Unit Trucks/Buses

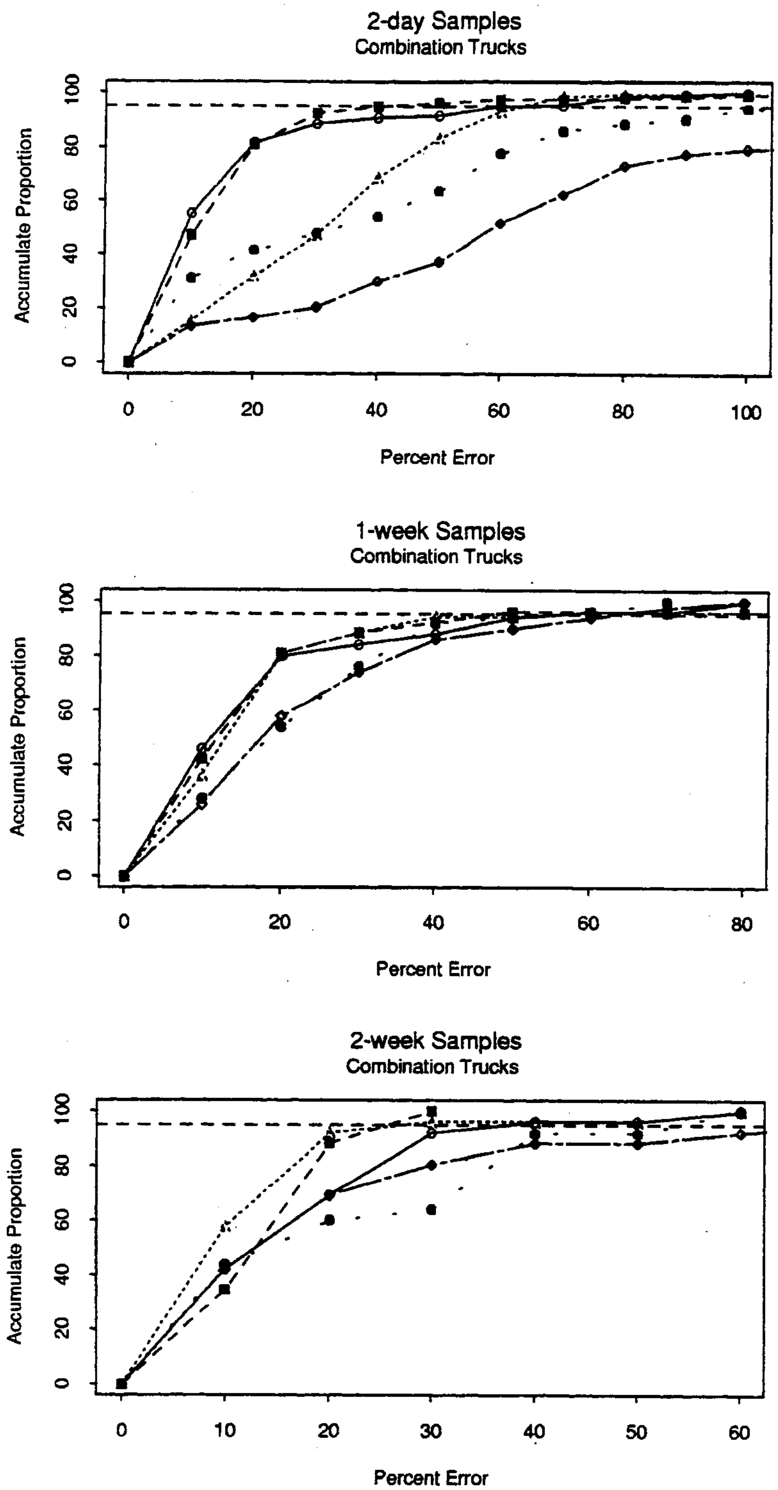


Figure A.5-c Plot of the Estimator Performance Summaries for Site 4037 (94) Combination Trucks

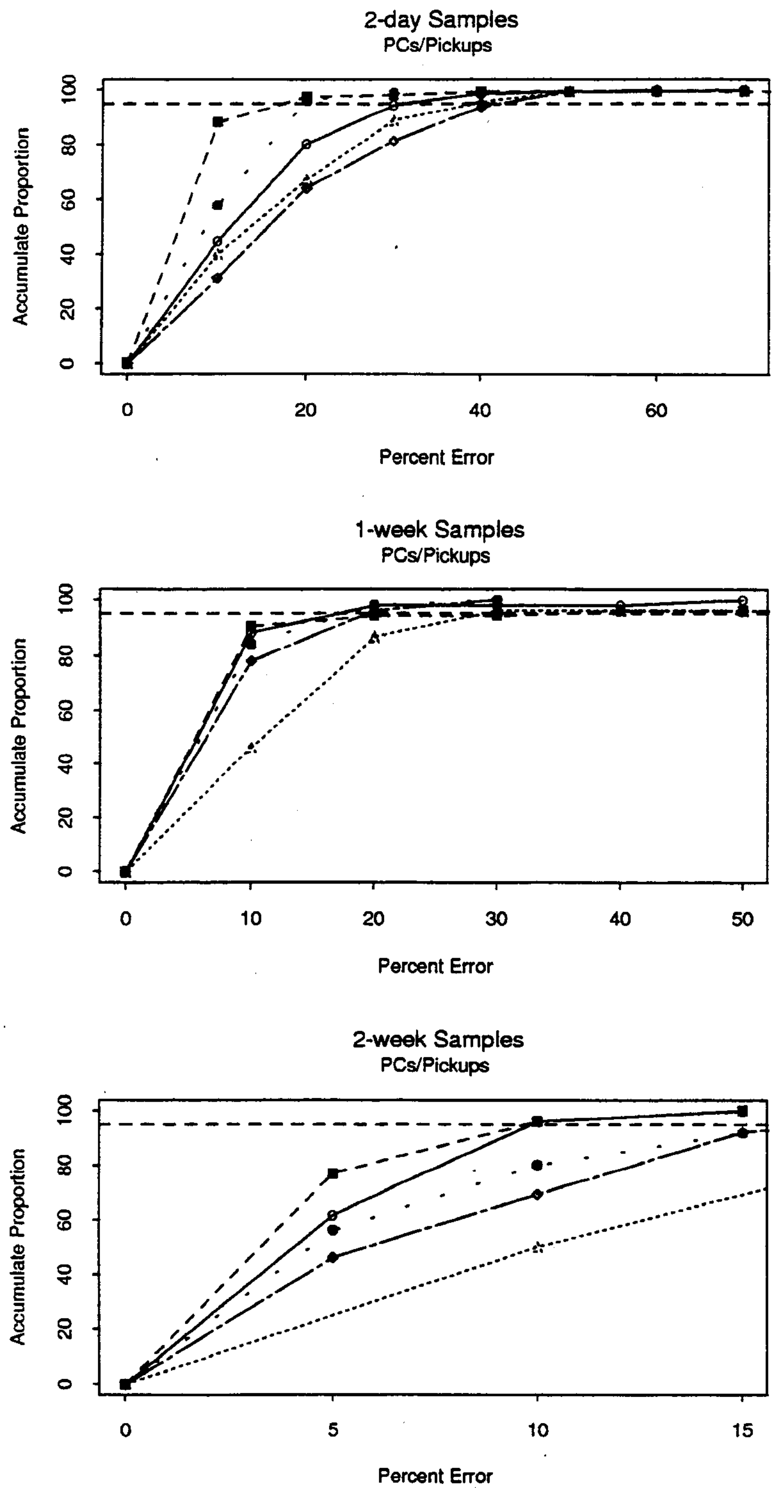


Figure A.6-a Plot of the Estimator Performance Summaries for Site 4055 (94) PCs/Pickups

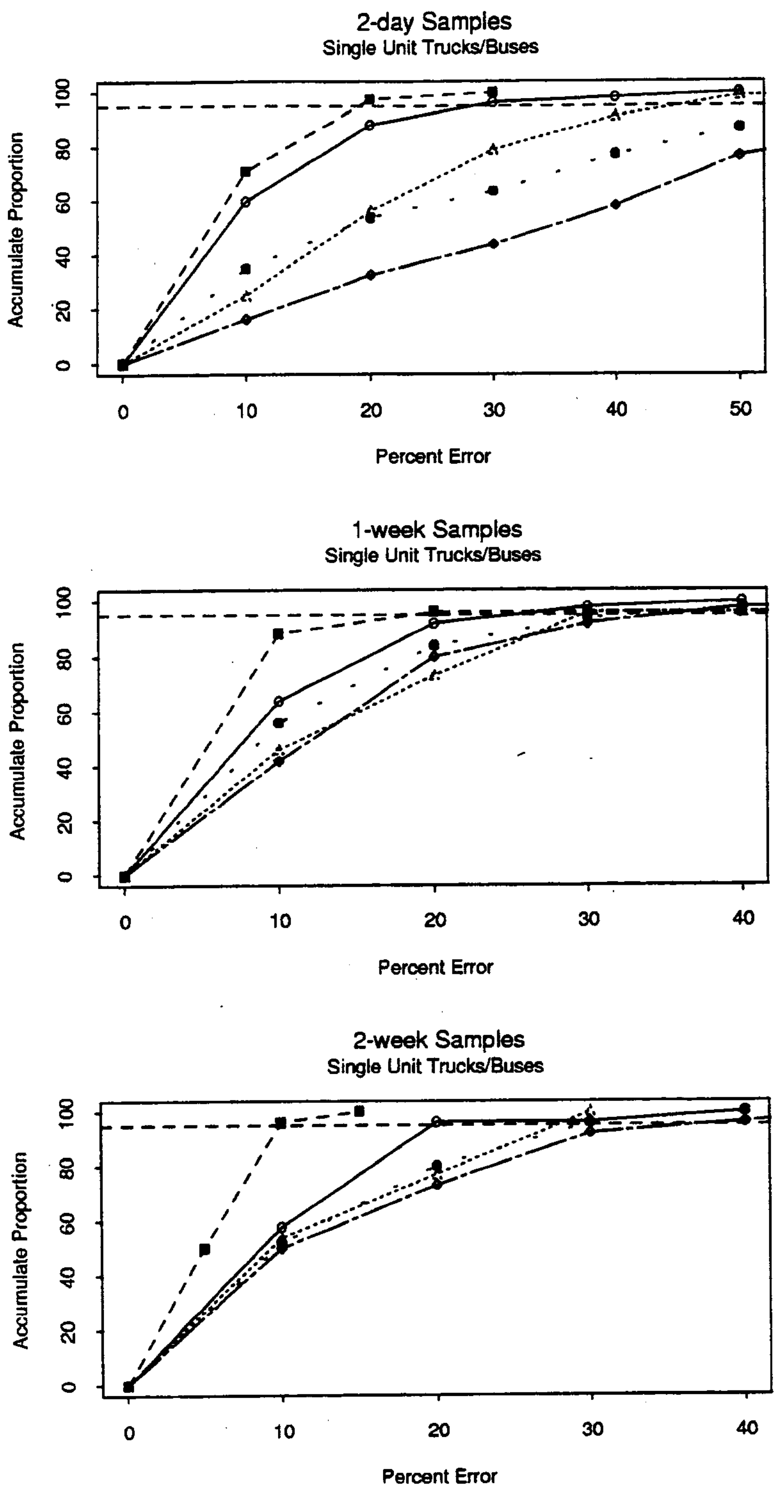


Figure A.6-b Plot of the Estimator Performance Summaries for Site 4055 (94) Single Unit Trucks/Buses

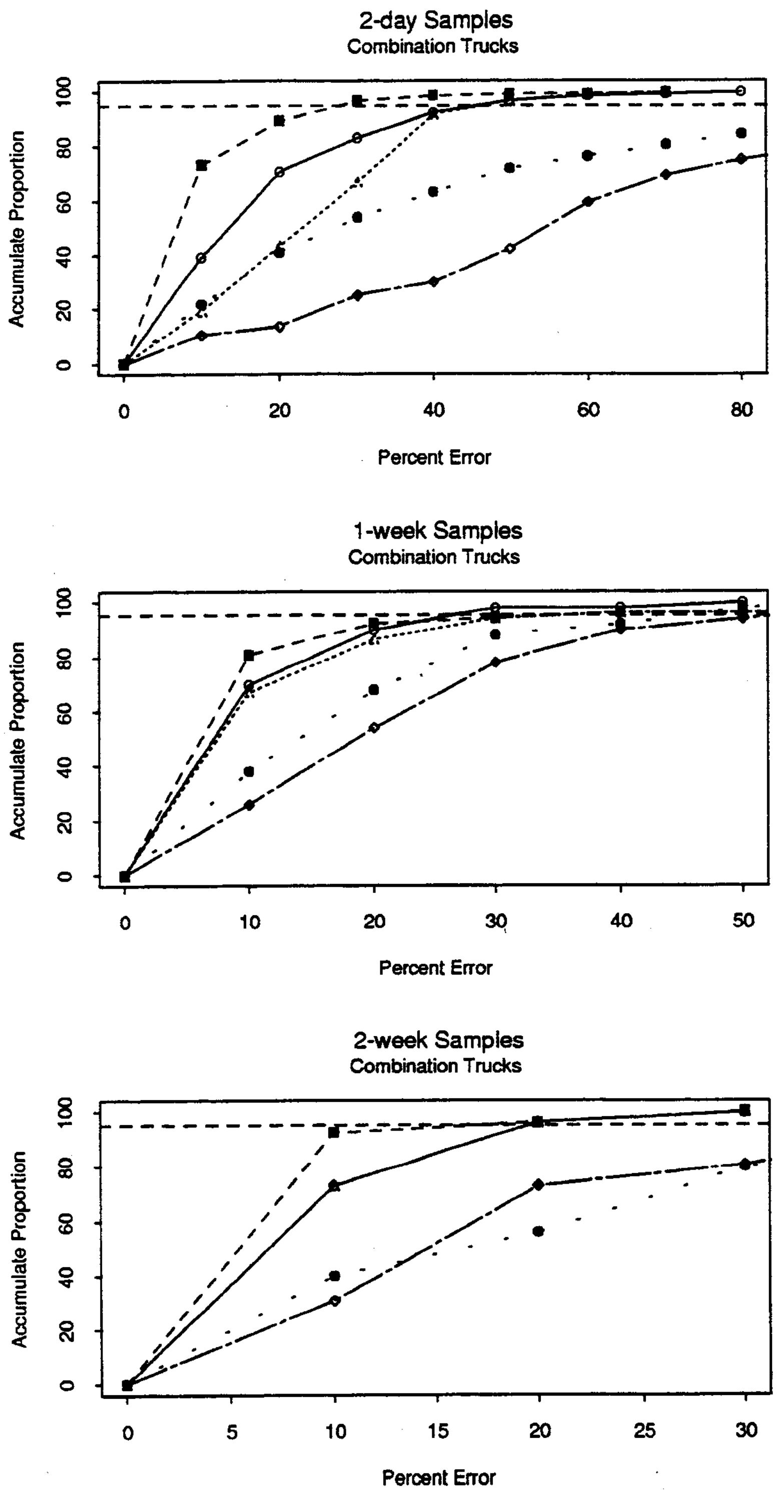


Figure A.6-c Plot of the Estimator Performance Summaries for Site 4055 (94) Combination Trucks

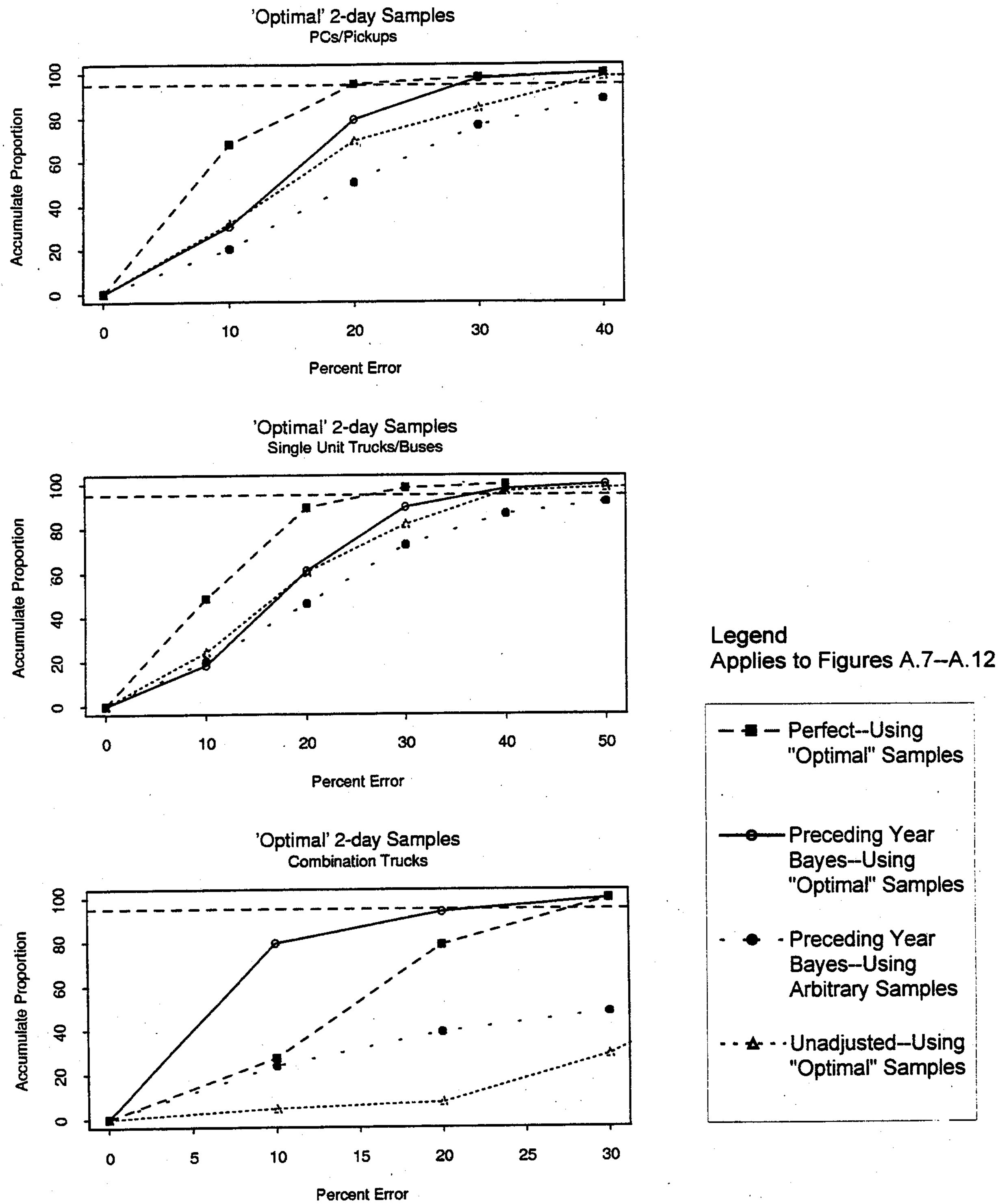


Figure A.7 Plot of the Estimator Performance Summaries for Site 1019 (94) "Optimal 2-day Samples"

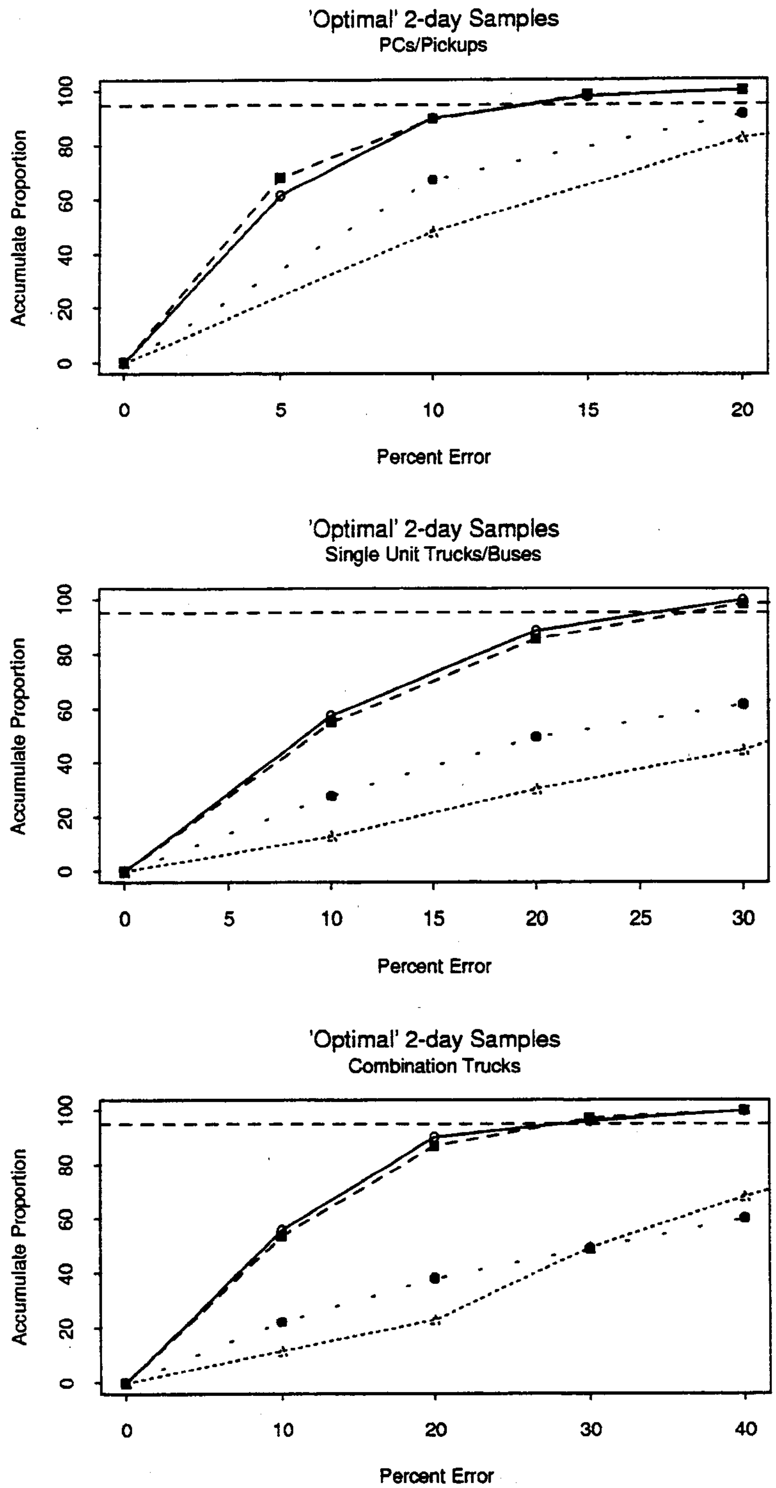


Figure A.8 Plot of the Estimator Performance Summaries for Site 1023 (94) "Optimal 2-day Samples"

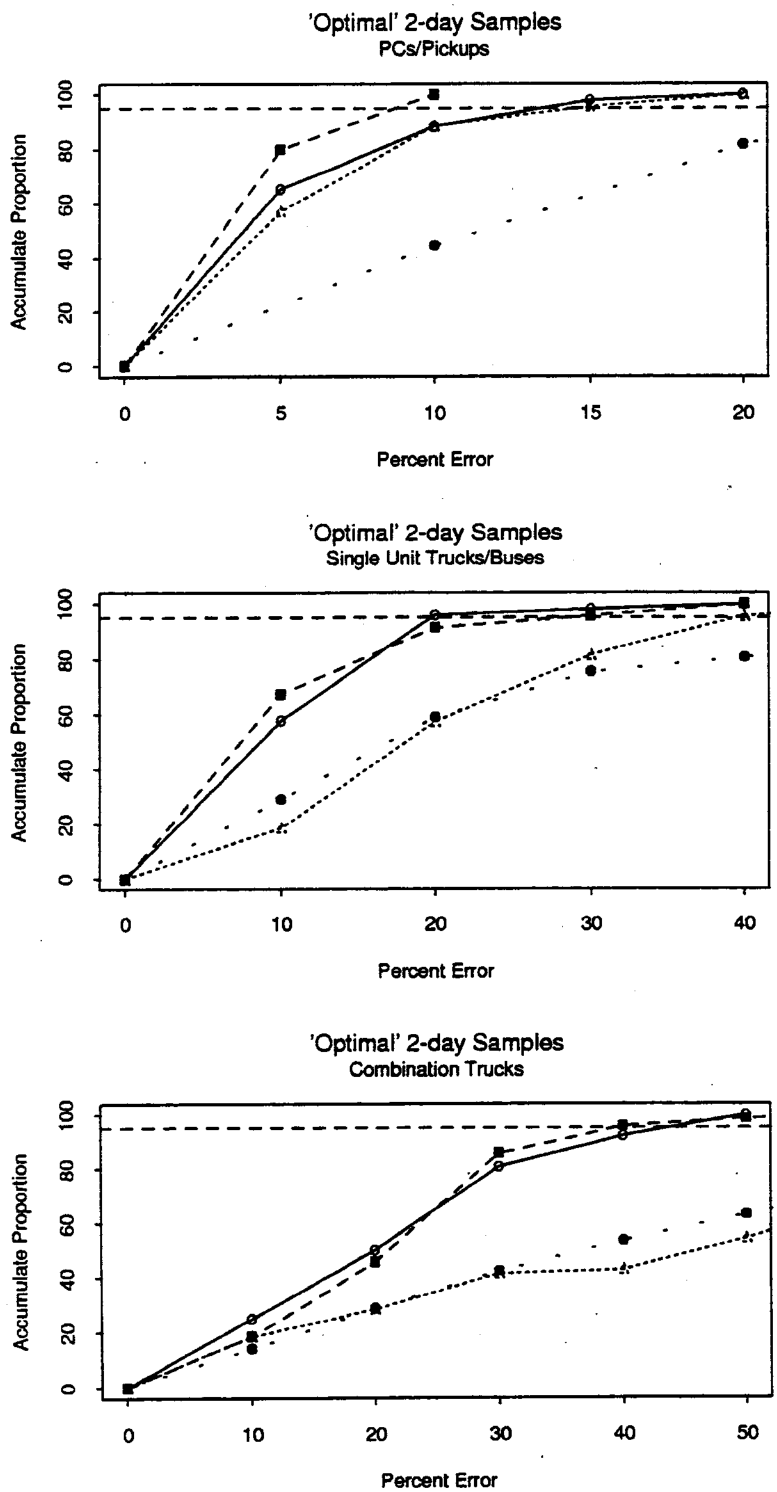


Figure A.9 Plot of the Estimator Performance Summaries for Site 1029 (94) "Optimal 2-day Samples"

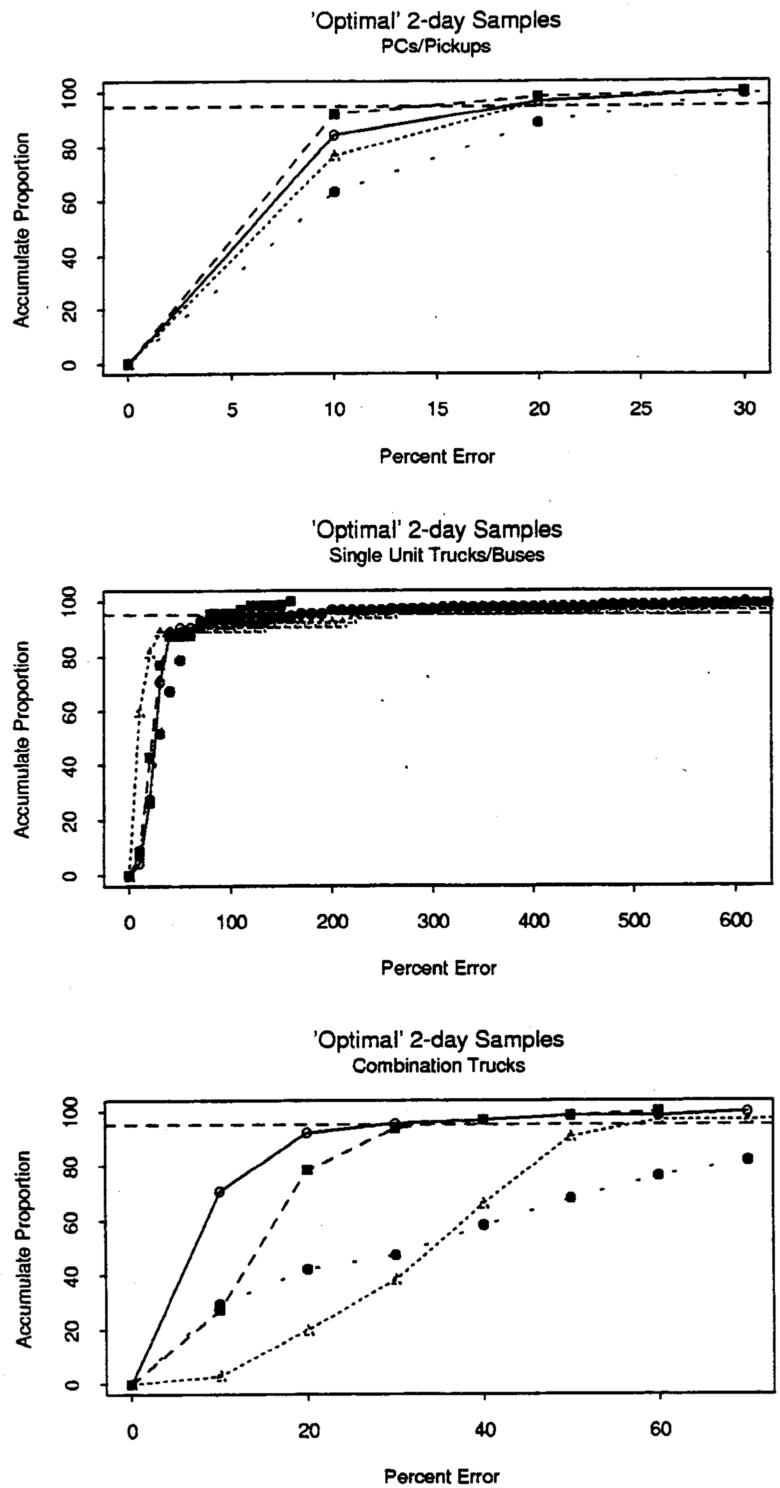


Figure A.10 Plot of the Estimator Performance Summaries for Site 4033 (94) "Optimal 2-day Samples"

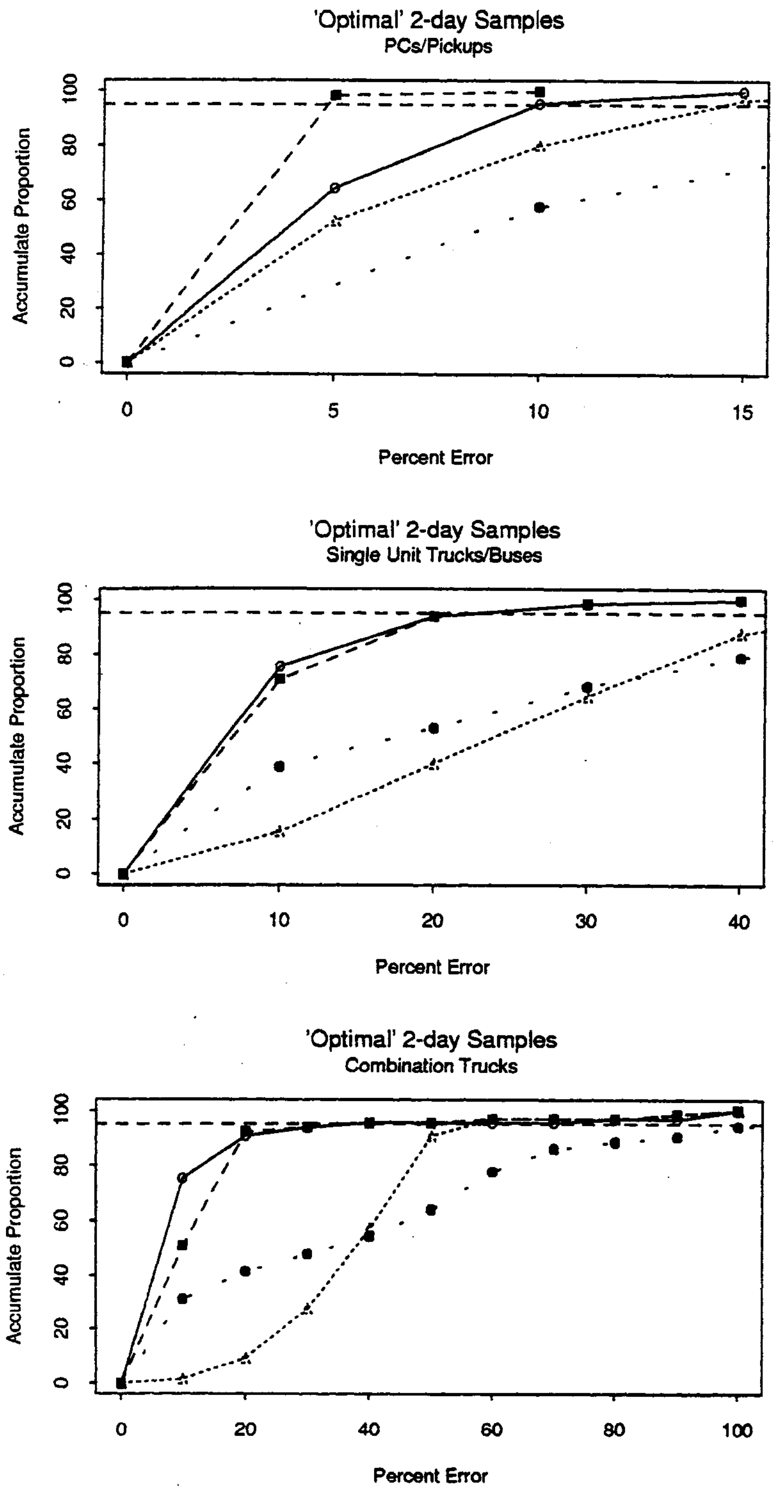


Figure A.11 Plot of the Estimator Performance Summaries for Site 4037 (94) "Optimal 2-day Samples"

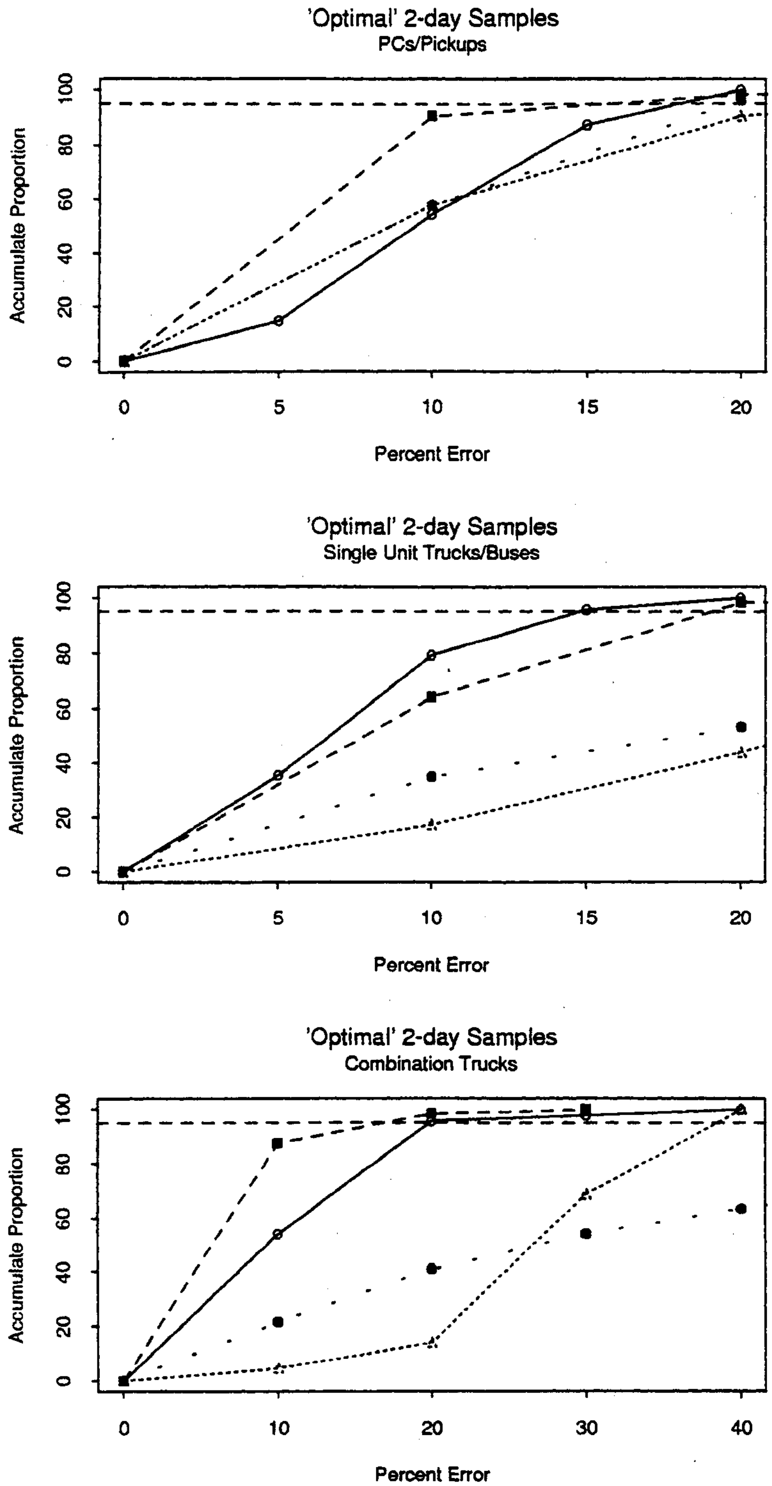


Figure A.12 Plot of the Estimator Performance Summaries for Site 4055 (94) "Optimal 2-day Samples"

APPENDIX B

S-PLUS PROGRAMS DEVELOPED FOR THE RESEARCH

I. S-Plus code *EBCLASM* to compute the posterior probabilities and the MDT

```
function(input.list, adjfac.list)
{
# input.list -- S-plus list containing the classification traffic count and the year, month and #
day in the month on which each count was collected
# adjfac.list -- S-plus list containing the factor group information needed to perform
#    Bayesian analysis
# pripi -- (NP×1) --prior probobility
# emPhi -- (3×(3×NP)) --empirical bayes priors for Phi
# emSigma -- (3×(3×NP)) --empirical bayes priors for Sigma
# beta1 -- ((3×17)× NP) estimates of adjustment factors for each factor group
# NS -- number of counts in the sample
# NP -- number of factor groups in the adjustment year
# N -- number of days in the sampling year (365/366)
#
        input <- input.list$input
        Year <- input.list$year
        beta1 <- adjfac.list$beta
        emPhi <- adjfac.list$Phi
        emSigma <- adjfac.list$Sigma
        pripi <- adjfac.list$Pripi
        adjyear <- adjfac.list$year
#
        library(Matrix)
        NP <- length(pripi)
        N <- 365
        if(leap.year(Year) == T)
                N <- 366
        NS <- length(input[, 1])
        y1 <- log(input[, 1:3])
        monindex <- input[, 4]
        dayindex <- input[, 5]
        ns2 <- 0
        pvec <- matrix(0, ncol = 1, nrow = NP)
        Mean <- matrix(0, ncol = 1, nrow = 3)
        Ones <- matrix(0, nrow = 3 * NS, ncol = 3)
        X1 <- matrix(0, nrow = 3, ncol = 17 * 3)
        X <- matrix(0, nrow = 3 * NS, ncol = 3 * 17)
        Y <- matrix(0, nrow = 3 * NS, ncol = 1)
        for(i in 1:NS) {
                ns2 <- ns2 + 1
                Y[((ns2 - 1) * 3 + 1):(ns2 * 3), 1] <- y1[i, ]
```

```
        Xm <- rep(0, 11)
        Xw <- rep(0, 6)
        if(monindex[i] == 12)
                Xm <- rep(-1, 11)
        else Xm[monindex[i]] <- 1
        if(dayindex[i] == 7)
                Xw <- rep(-1, 6)
        else Xw[dayindex[i]] <- 1
        for(j in 1:3)
                X1[j, ((j - 1) * 17 + 1):(j * 17)] <- cbind(t(Xm), t(Xw))
        X[((ns2 - 1) * 3 + 1):(ns2 * 3), ] <- X1
        Ones[((ns2 - 1) * 3 + 1):(ns2 * 3), ] <- diag(3)
}
for(i in 1:NP) {
        beta <- beta1[, i, drop = F]
        vmatout <- vmat(emPhi[, ((i - 1) * 3 + 1):(i * 3)], emSigma[, ((i - 1) * 3 +
        1):(i * 3)], input, Year)
        V <- vmatout$vm
        Gam0 <- vmatout$Gam0
        e1 <- Y - X %*% beta
        dx <- det(as.Matrix(V), logarithm = F)$modulus[1]
        Vi <- solve(V)
        SS <- t(Ones) %*% Vi %*% Ones
        SSinv <- solve(SS)
        Vvec <- as.matrix(diag(SSinv), ncol = 1, nrow = 3)
        A <- t(Ones) %*% Vi %*% e1
        U <- solve(SS, A)
        e2 <- e1 - Ones %*% U
        SS2 <- t(e2) %*% Vi %*% e2
        B <- det(as.Matrix(SS), logarithm = F)$modulus[1]
        mm <- as.vector(sqrt(dx * B) * exp(SS2/2))
        Gam <- as.matrix(diag(Gam0), ncol = 1, nrow = 3)
        Mean0 <- (exp(U + 1/2 * Gam + 1/2 * Vvec)/mm) * pripi[i]
        Mean <- Mean0 + Mean
        pvec[i, 1] <- 1/mm
}
den <- as.vector(crossprod(pripi, pvec))
mdt <- Mean/den
mprob <- pripi
mprob <- (pvec * pripi)/den
dimnames(mdt) <- list(c("PCs/Pickups", "SU Trucks", "Combo Trucks"),
c("MDT"))
list(Input = input.list, Adjyear = adjyear, Matchingprob = mprob, MDT = mdt)
```

```
}
```

II. S-Plus code *vmat* to compute the covariance matrix of the sample

```
function(Phi, Sigma, input, Year)
{
# calculate the covariance matrix (vm) of the observations in the classification count # sample
	monindex <- input[, 4]
	date <- input[, 6]
	dp <- matrix(NA, ncol = 1)
	NS <- length(monindex)
	ns <- 0
	for(i in 1:NS) {
		ns <- ns + 1
		dp[ns] <- julian(monindex[i], date[i], Year, origin = c(month = 1, day = 0,
		year = Year))
	}
	mx <- dp[ns] - dp[1]
	qq <- 0
	niter <- 0
	Gamin <- Sigma
	while(qq == 0) {
		Gamout <- Phi %*% Gamin %*% t(Phi) + Sigma
		if(sum((Gamout - Gamin) * (Gamout - Gamin)) <= 1e-020)
			qq <- 1
		else {
			Gamin <- Gamout
			niter <- niter + 1
		}
	}
	Gam0 <- Gamout
	vm <- matrix(0, ncol = 3 * ns, nrow = 3 * ns)
	for(i in 1:ns) {
		vm[(3 * (i - 1) + 1):(3 * i), (3 * (i - 1) + 1):(3 * i)] <- Gam0
	}
	if(ns > 1) {
		Gam <- matrix(0, ncol = 3, nrow = 3 * mx)
		Gam[1:3, ] <- Phi %*% Gam0
		if(mx > 1) {
			for(i in 2:mx)
				Gam[(3 * (i - 1) + 1):(3 * i), ] <- Phi %*% Gam[(3 * (i - 2)
				+1):(3 * (i - 1)), ]
```

```
                }
                for(i in 1:(ns - 1)) {
                        for(j in (i + 1):ns) {
                                a <- dp[j] - dp[i]
                                vm[(3 * (i - 1) + 1):(3 * i), (3 * (j - 1) + 1):(3 * j)] <- Gam[(
                                  3 * (a - 1) + 1):(3 * a), ]
                                vm[(3 * (j - 1) + 1):(3 * j), (3 * (i - 1) + 1):(3 * i)] <- Gam[(
                                  3 * (a - 1) + 1):(3 * a), ]
                        }
                }
        }
        list(vm = vm, Gam0 = Gam0)
}
```

APPENDIX C

ADDITIONAL ANALYSES CONDUCTED DURING THIS PROJECT

The body of this report describes a method for estimating classified MDT, based on modeling daily classified vehicle counts as the outcomes of a serially correlated multivariate lognormal distribution. Estimates of the monthly and day-of-week adjustments were computed using ordinary least-squares applied to the natural logarithms of the daily traffic counts, and estimates of the underlying time-series parameters were computed by applying least-squares to the resulting residuals. In the course of this project we also investigated the potential usefulness of an alternative estimation method, and of an alternative modeling approach, and this Appendix describes the results of these peripheral investigations.

Joint Estimation of Adjustment and Time-Series Parameters Via Maximum Likelihood

As noted earlier, the estimation methods described in this report can be viewed as multivariate generalizations of a method developed in an earlier Mn/DOT-sponsored project for estimating unclassified MDT from short traffic counts (Davis, 1997b). The earlier project also used a two-stage least-squares method to estimate monthly and day-of-week adjustment terms characterizing an ATR factor group, along with the time-series parameters characterizing the daily traffic counts at individual ATR sites. Now for some time it has been known that the log-likelihood function for regression/time-series models can be evaluated using the Kalman Filter, even when some of the data are missing (Harvey and Philips, 1979; Jones, 1980), and in fact this approach has been applied to daily freeway traffic counts to estimate the effects of interventions (Davis and Nihan, 1984). Since maximum likelihood estimates generally tend to be more statistically efficient than least-squares estimates, one of the first tasks of this current project was to assess the feasibility of replacing the two-stage least squares estimator by maximum likelihood.

To accomplish this, an S-PLUS routine was written which generalized the routine developed by Jones for computing the log-likelihood of a set of time-series parameters to the situation where we have several time-series all having the same regression parameters, but each having a (possibly) different time-series model for its residuals. In essence this required applying a separate Kalman

Filter transformations to the dependent and independent variables for each time-series, and then combining these to compute the overall likelihood. This routine was then embedded in the S-PLUS routine *nlminb*, a quasi-Newton method for computing the minima of nonlinear functions. To test the routine, computational experiments using daily traffic counts from (nonclassifying) ATRs from the year 1991 were used to compute maximum likelihood and least-squares estimates.

Two experiments were carried out in order to compare the results using the above two methods.

First Experiment

Test Dataset

Traffic count data from ATR station 1 in the year 1991 and from ATR station 220 in 1991. These both belonged to Mn/DOT factor group 4.

Computation Time

- Maximum Likelihood method

  About 16 hours were needed to run the nonlinear optimization routine in a Pentium 200 MHz computer and another 9 minutes was required to solve for the linear regression coefficients.

- Two-Stage Least Squares method

  Total evaluation time including preparing the data for Minitab, saving the first stage residuals and then retrieving them for being analyzed in S-Plus, did not exceed 1 hour.

Second Experiment

Test Dataset

Traffic count data from stations 1,51,55 and 57, in factor group 4, for the year 1991.

Computation Time

- Maximum Likelihood method

  Increasing the number of stations from 2 to 4 increased the number of AR parameters to be estimated from 4 to 8. Estimation of these required 77 hours and 45 minutes in a Pentium 200 MHz computer. And another 34 minutes was needed to get the linear regression coefficients.

- Two-Stage Least Squares method

Increasing the station number did not dramatically alter the evaluation time to find the linear regression parameters and the time series parameters.

The estimation results for these two experiments are listed in Table C.1. Estimates from two-stage least squares are listed under LS and those from maximum likelihood method are under ML column. The parameters including the estimates of the natural logarithm of the mean daily traffic and the monthly and day-of-week adjustment terms, estimates of the time series parameters and the variances of the innovation terms.

From the tables, we can see that both methods produce similar fits. The estimates of the natural logarithm of the mean daily traffic and the monthly and day-of-week adjustment factors obtained by these two methods are very close. And the variances of the innovations are low for both methods.

A comparison of the difference between the maximum likelihood estimates and the least squares estimates can be made using Absolute Percent Error (APE),which is defined as:

$$\text{APE} = |\text{PE}| = \left| \frac{LS \quad Estimate \; - ML \quad Estimate}{ML \quad Estimate} \times 100 \right|$$

The APE values for the mean daily traffic and the adjustment factors are listed in Table C.2. These small APE values mean the two-stage least squares estimates and the maximum likelihood estimates are very close. Almost all APEs are less than 6%. For the first experiment, the APE of more than half of the adjustment factors are less than 3% and for the second experiment, only 5 adjustment factors have APE greater than 3%. For the MDT in the order of 1500-2000, 3% difference will only lead to a difference of traffic volume of about 40--60 vehicles per day.

As a conclusion, the differences between the maximum likelihood estimates and the least squares estimates are not sufficient to support very large increase in computational effort needed to carry out the ML approach.

Table C.1 Two-Stage Least Squares Estimates and Maximum Likelihood Estimates from Two Experiments

First Experiment			Second Experiment		
	LS Estimates	ML Estimates	LS Estimates	ML Estimates	
MDT-s1	7.43239	7.42544	7.43347	7.42850	**MDT-s1**
MDT-s220	7.46887	7.45905			
			5.98723	6.00584	**MDT-s51**
			6.40655	6.39910	**MDT-s55**
			7.12859	7.12704	**MDT-s57**
m1	-.35478	-.33168	-.38540	-.37831	**m1**
m2	-.30329	-.24800	-.33533	-.31827	**m2**
m3	-.29020	-.27240	-.36736	-.32997	**m3**
m4	-.15238	-.13090	-.15909	-.13678	**m4**
m5	.05087	.07808	.17912	.16843	**m5**
m6	.32562	.29004	.41788	.36527	**m6**
m7	.47318	.41543	.52334	.48319	**m7**
m8	.46725	.39976	.48535	.44401	**m8**
m9	.21499	.20546	.17609	.19934	**m9**
m10	.03398	.07215	.04861	.09109	**m10**
m11	-.20043	-.25316	-.26155	-.27798	**m11**
m12	-.26481	-.22478	-.32166	-.31002	**m12**
w1	-.07720	-.08257	-.07548	-.05792	**w1**
w2	-.02832	-.03520	-.04484	-.05521	**w2**
w3	-.04506	-.04248	-.03526	-.04016	**w3**
w4	-.05420	-.04266	-.03200	-.02939	**w4**
w5	-.02703	-.01833	-.02414	-.02172	**w5**
w6	.20847	.20759	.13827	.13955	**w6**
w7	.02334	.01365	.07345	.06485	**w7**
Φ_1-s1	.43576	.37286	.47754	.37273	**Φ_1-s1**
Φ_7-s1	.55645	.18132	.60290	.18121	**Φ_7-s1**
Φ_1-s220	.64237	.65480			
Φ_7-s220	.44817	.23760			
			.72913	.60404	**Φ_1-s51**
			.41649	.28485	**Φ_7-s51**
			.53880	.51792	**Φ_1-s55**
			.21625	.19034	**Φ_7-s55**
			.50608	.56955	**Φ_1-s57**
			.22608	.00983	**Φ_7-s57**
σ^2-s1	.01925	.01684	.01994	.01685	**σ^2-s1**
σ^2-s220	.02205	.01865			
			.02421	.02312	**σ^2-s51**
			.01383	.01431	**σ^2-s55**
			.03195	.02731	**σ^2-s57**

Table C.2 Comparison of the Two-Stage LS Estimates and the ML Estimates

Comparison of the LS Estimates and the ML Estimates							
	First Experiment			Second Experiment			
	LS	ML	APE	LS	ML	APE	
MDT-s1	1689.8	1678.1	0.70	1691.7	1683.3	0.50	MDT-s1
MDT-s220	1752.6	1735.5	0.99				
				398.3	405.8	1.84	MDT-s51
				605.8	601.3	0.75	MDT-s55
				1247.1	1245.2	0.16	MDT-s57
m1	0.701	0.718	2.28	0.680	0.685	0.71	m1
m2	0.738	0.780	5.38	0.715	0.727	1.69	m2
m3	0.748	0.762	1.76	0.695	0.719	3.32	m3
m4	0.859	0.877	2.13	0.853	0.872	2.21	m4
m5	1.052	1.081	2.68	1.196	1.183	1.08	m5
m6	1.385	1.336	3.62	1.519	1.441	5.40	m6
m7	1.605	1.515	5.95	1.688	1.621	4.10	m7
m8	1.596	1.491	6.98	1.625	1.559	4.22	m8
m9	1.240	1.228	0.96	1.193	1.221	2.30	m9
m10	1.035	1.075	3.75	1.050	1.095	4.16	m10
m11	0.818	0.776	5.41	0.770	0.757	1.66	m11
m12	0.767	0.799	3.92	0.722	0.733	1.52	m12
w1	0.926	0.921	0.54	0.927	0.944	1.74	w1
w2	0.972	0.965	0.69	0.956	0.946	1.04	w2
w3	0.956	0.958	0.26	0.965	0.961	0.49	w3
w4	0.947	0.958	1.15	0.969	0.971	0.26	w4
w5	0.973	0.982	0.87	0.976	0.979	0.24	w5
w6	1.232	1.231	0.09	1.148	1.150	0.13	w6
w7	1.024	1.014	0.97	1.076	1.067	0.86	w7

Note: APE entries are percentages

Alternative Model of Daily Classified Traffic Volumes

The lognormal model of daily traffic volumes recommends itself primarily because the formidable body of statistical techniques developed for normally distributed observations can be brought to bear on the logarithms of the traffic counts. For some vehicle classes however, such as double- or triple-trailer combinations, it may be that no vehicles are observed on some days and since the logarithm of zero is undefined, the lognormal model will not be strictly applicable. Because it may be important nonetheless to predict volumes for these more rare vehicle classes, it was decided as part of this project to explore an alternative statistical model that allowed for some zero daily counts.

The alternative model had a hierarchical structure with three layers, as follows. (1) Given the total traffic volume for a day at a site, the traffic volumes in the vehicle classes were modeled as multinomial outcomes whose probabilities were allowed to vary with the day of the week and the month of the year via a multinomial logit model. (2) Given the expected total daily traffic at the site, the actual traffic volume was modeled as a Poisson outcome. (3) The natural logarithm of the expected daily total traffic volume was modeled as the outcome of an autoregressive process with a mean value which varied with the day of the week and with the month, similar to that used in the lognormal model described in the body of this report. Layer (1) is a natural model for distributing an integer-valued total count among vehicle classes, while layers (2) and (3) produce integer-valued total daily counts which are (a) over-dispersed relative to the basic Poisson model and (b) are serially correlated. As noted in the body of the report, over-dispersion and serial correlation are statistical properties which often appear in daily traffic count data.

To investigate the plausibility of the alternative model, two statistical analyses were performed. In the first analysis, the daily count data for the year 1992 from LTPPP site 1085 were first aggregate into four vehicle classes, as follows:

Class 1: Passenger cars, pick-up trucks and vans,

Class 2: Single-unit trucks and buses,

Class 3: Single-trailer combination vehicles,

Class 4: Multi-trailer combinations vehicles.

Letting $p_{t,k}$ denote the probability a randomly chosen vehicle on day t belonged to class k, these probabilities were modeled using a multinomial logit model, of the form

$$\text{logit}\ (p_{t,k}) = \mu_k + \sum_{j=1}^{12} \delta_{t,j} m_j + \sum_{i=1}^{7} \Delta_{t,i} w_i$$

Maximum likelihood estimates of the μ, m and w coefficients were computed by treating the total traffic volume on each as a given quantity, and for each vehicle class the observed and predicted number of days during 1992 showing a given number of vehicles were computed and compared using a χ-squared goodness-of-fit test. Figure C.1 summarizes the results of these tests, and it can be seen that for the first three vehicle classes the predicted distribution gives a satisfactory fit to the observed distribution. For the fourth vehicle class, multi-trailer combination vehicles,the fit is less satisfactory, but this is not surprising since the low numbers of vehicles in this class means that its contribution to the overall likelihood is small. It was thus concluded that over the multinomal logit model provided a plausible description of how vehicles were distributed among classes, but actual application of the model may require a separate estimation for the low volume vehicle class.

The second analysis looked into the plausibility of layers (2) and (3),again using count data for 1992 from LTPPP site 1085, but now with the total daily traffic count as a dependent variable. We will refer to this model as a Poisson-lognormal (PLN) model. Unlike the layer (1) conditional multinomial model, estimation of the PLN model poses some technical difficulties, due to the fact that the layer (3) component is not observed directly. Chan and Ledolter [30] review the difficulties in fitting models of this type, and describe a method for maximum likelihood estimation based on the EM algorithm, but where the expectation step is carried out using a simulation technique called Gibbs sampling. An alternative would be to take a fully Bayesian approach, and use Gibbs sampling to estimate the posterior distributions of the quantities of interest, as outlined in Spiegelhalter *et al* [31]. This second approach would require only a few runs of the Gibbs sampler rather than the potentially very large number required by Chan and Ledolter's method, and was the approach selected for this analysis.

A reasonably general computer program for implementing Gibbs sampling, called BUGS, has been developed at the Biostatistics Unit at Cambridge University and this program along with supporting documentation was downloaded from the developer's website. A BUGS input file describing the model outlined in layers (2) and (3) was constructed, and it and the resulting output of a typical run are displayed in Figure C.2. After numerous experiments with this data set, the following conclusions were reached:

(I) The Gibbs sampler output for the monthly and day-of-week adjustment factors tended to show a high degree of serial correlation, meaning that fairly long runs (requiring 30 or more minutes on a 100 MHz Pentium computer) were needed to estimate these quantities reliably;

(II) Assessing the convergence of the Gibbs sampler requires some knowledge of time-series analysis and Markov chain theory;

(III)Although it was possible to get BUGS to run when the input counts had no missing data, it was not possible to get it to accept missing data.

None of these limitations is necessarily fatal to use of Gibbs sampling. Computers continue to become faster, and some progress has been made toward a more automatic testing for Gibbs sampler convergence (Best, Cowls and Vines, 1996). As to (III) above, in principle missing data could be accommodated by developed a specialized Gibbs sampler computer program (Chan and Ledolter, 1995). However, it was concluded that the additional effort needed to turn this approach into a practical methodology for working with classification count data would likely exceed the resources available to this project. We nonetheless see this as a very promising topic very further research.

Figure C.1 Conditional Multinomial Fits, using Site 1085 (92)

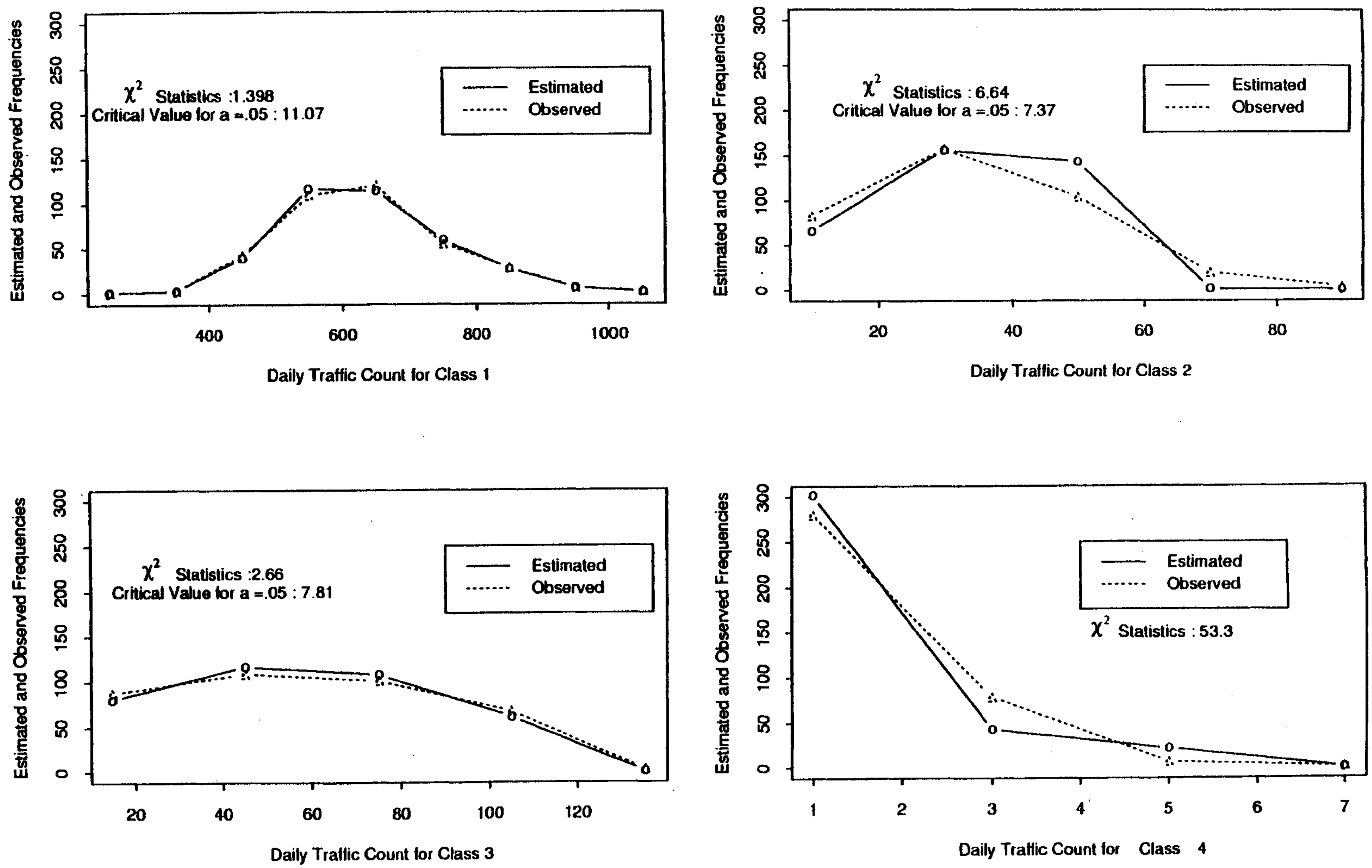


Figure C.2 Example BUGS Run

```
Welcome to BUGS on 27 th May 1998  at 17:21:34
BUGS : Copyright © 1992 .. 1995 MRC Biostatistics Unit.
All rights reserved.
Version 0.600 for 32 Bit PC.
For general release: please see documentation for disclaimer.
The support of the Economic and Social Research Council (UK)
is gratefully acknowledged.
Bugs>compile("lgnpoi4.bug")
model lgnpoi3;
const
        N=366;
var
        y[N], month[N], day[N], nohol[N], ebar[N], e[N], mu[N],mf[12],wf[7],phi,u,tau,sigma;
data y, month, day, nohol in "cnt8592.dat";
inits in "c85925k4.in";
{
ebar[1] <- 0;
e[1] <- 0;
for (i in 2:N) {
  ebar[i] <- phi*e[i-1];
  e[i]~dnorm(ebar[i],tau);
 }
for (i in 1:N) {
  log(mu[i]) <- u+mf[month[i]]+wf[day[i]]+e[i];
  y[i]~ dpois(mu[i]);
 }
sigma <- 1.0/sqrt(tau);
u ~ dnorm(0.0,1.0e-04);
tau ~ dgamma(1.0e-03,1.0e-03);
phi ~ dnorm(0.0, 1.0e-04);
for (j in 1:11) {  mf[j]~dnorm(0.0,1.0e-04);}
mf[12] <- (-1)*sum(mf[1:11]);
for (j in 1:6)   { wf[j]~ dnorm(0.0, 1.0e-04);}
wf[7] <- (-1)*sum(wf[1:6]);
}

Parsing model declarations.
Loading data value file(s).
Loading initial value file(s).
Parsing model specification.
```

Checking model graph for directed cycles.
Possible directed cycle or undirected link in model
Generating code.
Generating sampling distributions.
Checking model specification.
Choosing update methods.
compilation took 00:00:02
Bugs>monitor(u)
Bugs>monitor(mf)
Bugs>monitor(wf)
Bugs>monitor(phi)
Bugs>monitor(sigma)
Bugs>monitor(mu)
Bugs>update(1000) time for 1000 updates was 00:05:14
Bugs>diag(u)

	mean	sd	mean	sd	Z	sample
	6.56	2.91E-5	6.56	2.32E-5	-9.14E-1	1000

Bugs>diag(mf)

	mean	sd	mean	sd	Z	sample
[1]	-2.91E-1	1.52E-4	-2.84E-1	1.67E-4	-1.77	1000
[2]	-1.74E-1	2.09E-4	-1.76E-1	2.12E-4	5.24E-1	1000
[3]	-1.51E-1	2.07E-4	-1.61E-1	2.33E-4	2.16	1000
[4]	-1.47E-2	2.32E-4	-1.54E-2	2.63E-4	1.56E-1	1000
[5]	1.12E-1	2.82E-4	1.11E-1	1.83E-4	1.66E-1	1000
[6]	1.57E-1	3.89E-4	1.46E-1	3.95E-4	1.96	1000
[7]	1.17E-1	2.62E-4	1.27E-1	3.60E-4	-2.01	1000
[8]	1.77E-1	1.27E-4	1.87E-1	2.10E-4	-2.90	1000
[9]	1.48E-1	2.66E-4	1.39E-1	2.65E-4	1.87	1000
[10]	1.68E-1	6.23E-4	1.68E-1	3.76E-4	2.14E-2	1000
[11]	-5.46E-2	2.75E-4	-4.85E-2	2.76E-4	-1.29	1000
[12]	-1.95E-1	2.67E-4	-1.95E-1	1.91E-4	1.65E-1	1000

Bugs>diag(wf)

	mean	sd	mean	sd	Z	sample
[1]	-2.56E-2	6.38E-5	-2.27E-2	7.04E-5	-1.24	1000
[2]	-6.71E-2	4.99E-5	-6.77E-2	4.75E-5	3.33E-1	1000
[3]	-2.50E-2	6.54E-5	-2.50E-2	3.20E-5	-1.91E-2	1000
[4]	-2.88E-2	7.56E-5	-2.98E-2	6.55E-5	3.84E-1	1000
[5]	1.14E-2	7.83E-5	1.05E-2	5.33E-5	3.83E-1	1000
[6]	1.28E-1	1.02E-4	1.28E-1	8.31E-5	1.98E-2	1000
[7]	6.63E-3	7.79E-5	6.20E-3	4.20E-5	1.96E-1	1000

Bugs>diag(phi)

	mean	sd	mean	sd	Z	sample
	3.16E-1	5.83E-4	3.13E-1	2.73E-4	4.00E-1	1000

```
Bugs > diag(sigma)
            mean        sd        mean        sd        Z        sample
          8.35E-2   3.99E-6   8.33E-2   1.76E-6   3.84E-1      1000
Bugs > stats(u)
            mean          sd        2.5% : 97.5% CI      median       sample
          6.562E+0   6.056E-3   6.549E+0   6.573E+0   6.562E+0     1000
Bugs > stats(mf)
            mean          sd        2.5% : 97.5% CI      median       sample
[1]       -2.865E-1   1.928E-2  -3.249E-1  -2.453E-1  -2.873E-1     1000
[2]       -1.760E-1   1.864E-2  -2.120E-1  -1.426E-1  -1.752E-1     1000
[3]       -1.558E-1   2.062E-2  -1.929E-1  -1.155E-1  -1.564E-1     1000
[4]       -1.858E-2   2.181E-2  -5.927E-2   3.061E-2  -1.939E-2     1000
[5]        1.096E-1   1.978E-2   6.954E-2   1.439E-1   1.105E-1     1000
[6]        1.479E-1   2.334E-2   1.021E-1   1.985E-1   1.489E-1     1000
[7]        1.307E-1   1.993E-2   8.756E-2   1.676E-1   1.319E-1     1000
[8]        1.904E-1   1.868E-2   1.570E-1   2.274E-1   1.888E-1     1000
[9]        1.397E-1   2.096E-2   1.001E-1   1.814E-1   1.388E-1     1000
[10]       1.694E-1   2.242E-2   1.283E-1   2.199E-1   1.696E-1     1000
[11]      -4.872E-2   1.805E-2  -8.332E-2  -1.248E-2  -4.922E-2     1000
[12]      -2.021E-1   2.201E-2  -2.474E-1  -1.607E-1  -2.013E-1     1000
Bugs > stats(wf)
            mean          sd        2.5% : 97.5% CI      median       sample
[1]       -2.622E-2   1.256E-2  -5.076E-2  -1.433E-3  -2.623E-2     1000
[2]       -6.905E-2   1.101E-2  -9.045E-2  -4.853E-2  -6.874E-2     1000
[3]       -2.175E-2   1.073E-2  -4.074E-2   2.559E-4  -2.238E-2     1000
[4]       -2.603E-2   1.153E-2  -4.878E-2  -3.028E-3  -2.539E-2     1000
[5]        1.136E-2   1.017E-2  -8.578E-3   3.114E-2   1.119E-2     1000
[6]        1.288E-1   1.137E-2   1.077E-1   1.500E-1   1.291E-1     1000
[7]        2.812E-3   1.196E-2  -2.087E-2   2.485E-2   3.107E-3     1000
Bugs > stats(phi)
            mean          sd        2.5% : 97.5% CI      median       sample
          3.150E-1   6.216E-2   1.919E-1   4.337E-1   3.103E-1     1000
Bugs > stats(sigma)
            mean          sd        2.5% : 97.5% CI      median       sample
          8.351E-2   3.909E-3   7.618E-2   9.139E-2   8.343E-2     1000
Bugs > q()
```